

Long-Run Implications of Alternative Emission Trading Plans: An Experiment with Robot Traders

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Abstract

Two approaches to emissions trading are cap-and-trade, in which an aggregate cap on emissions is distributed in the form of permits, and baseline-and-credit, in which firms earn credits for emissions below their baselines. Theoretical considerations suggest the long-run equilibria of the two plans will differ if baselines are proportional to output, because a variable baseline is equivalent to an output subsidy. This paper reports on a laboratory environment designed to test this prediction. A computerized environment has been created in which subjects representing firms choose capacity and emission rates and participate in markets for permits or credits and for output. Demand for output is simulated. All decisions are tracked through a double-entry bookkeeping system. The timing of decisions was adjusted to avoid short-run instability. The paper reports the parameterization for an experiment with human traders and results of a simulated experiment using robots.

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Introduction

There are two distinct approaches to emissions trading: cap-and-trade or baseline-and-credit. Under a cap-and-trade plan an aggregate cap is placed on emissions of the relevant firms. A corresponding quantity of emission permits (often called *allowances*) is created. The permits may be sold at auction or distributed to incumbent firms. Firms must surrender a unit of allowances for every unit of emissions discharged over a time period. Firms may sell allowances that they expect not to use or purchase allowances to cover emissions in excess of the original distribution. Under a baseline-and-credit plan firms are assigned a baseline emissions level. If their actual emissions are below the baseline, they earn *emission reduction credits*. These credits may be sold to firms whose emissions exceed their baselines.

The two plans are theoretically equivalent if the baseline in a baseline-and-credit plan is numerically equal to the allowances received under a cap-and-trade plan. In many cases, however, the baseline is related to the capacity or output of the regulated firms. For example, the baseline may be computed by multiplying output by a prescribed performance standard which specifies the target industry emission rate. In this case theoretical considerations suggest the long-run equilibria of the two plans will differ because a variable baseline is equivalent to an output subsidy. Compared to a cap-and-trade plan with the same average emission rate, the baseline-and-credit plan will exhibit higher output and emissions. Compared to a cap-and-trade plan with the same emissions, the baseline-and-credit plan will exhibit a lower (and more costly) emission rate. Thus baseline-and-credit plans entail an inherent efficiency loss even before considering costs of administration.

Although the theoretical prediction is reasonably straightforward it relies on competitive equilibria being realized in two interrelated markets: the market for output and the market for emission permits or credits. Although some market institutions (such as the double auction) are highly effective in achieving equilibrium in a single market, it is less evident that competitive markets can achieve efficient outcomes when firms must optimize in two or more markets. If the theoretical prediction is not to be considered a mere curiosity, it would be useful to demonstrate its existence in real markets. Laboratory markets are ideal for this purpose. They can be made to reflect a substantial level of institutional detail while exerting careful control over a wide range of factors which are uncontrolled in a natural setting. This use of laboratory markets is frequently called "testbedding".

This paper reports progress on a laboratory experiment designed to testbed basic forms of the cap-and-trade and the baseline-and-credit methods of emissions trading and more particularly, to test whether the predicted difference in emissions levels will actually be realized. The report is

preliminary: no sessions with human subjects have yet been run. Instead the paper focuses on a sophisticated computerized environment which has been created to conduct the experiment. The paper is organized as follows. First, we describe the theoretical predictions in more detail. Second, we describe the computerized trading environment which we have implemented. Third, we describe the strategies followed by robot subjects that we have used to test the program. Fourth we report the results of some initial simulations. We conclude with discussion and speculation.

Theoretical Analysis

In this section we demonstrate that long-run equilibrium emissions and output are higher under a baseline-and-credit plan than under a cap-and-trade plan with the same industry-average emission rate. We confine attention to the case implemented in our experiment, in which firms producing under conditions of constant cost compete in both the market for emission rights and for output. Full details are given in Appendices A and B. The notation is summarized in Table 1.

Table 1 about here

Consider an industry with N firms. Each firm $i \in [1, \dots, N]$ produces q_i units of output and e_i units

of emissions, thus achieving an emission rate of $r_i = \frac{e_i}{q_i}$. Industry output is $Q = \sum_{i=1}^N q_i$. Aggregate

emissions are $E = \sum_{i=1}^N e_i = \sum_{i=1}^N r_i q_i$. Environmental damages are a positive and weakly convex

function of total emissions, $D = D(E)$, $D'(E) > 0$, $D''(E) \geq 0$. Willingness-to-pay for the output is a

weakly concave function of aggregate output, $WTP = \int_0^Q P(z) dz$, where $P = P(Q)$ is an inverse

demand curve with positive ordinate $P(0) > 0$ and negative slope $P'(Q) \leq 0$. The private cost of production is a linear homogenous function of output and emissions: $C_i = C_i(q_i, e_i)$. We separate

cost into unit capital cost $c_i(r_i)$, which is a declining function of the emission rate,

$c_i(r_i) > 0$, $c_i'(r_i) \leq 0$, and unit variable cost $w_i q_i$, which is a constant function of output.

Consequently total cost $C_i = c_i(r_i) q_i + w_i q_i$. Note that the marginal cost of output is $c_i(r_i) + w_i$ and

the marginal cost of abating pollution is $MAC = -\frac{\partial C_i}{\partial e_i} = -c_i'(r_i) > 0$.

Below we illustrate the optimal case in comparison to the two trading scheme outcomes. For details on predictions regarding the uncontrolled equilibrium for our model see Appendices A and B.

Social Optimum

An omnipotent social planner would choose an emission rate and output for each firm in order to maximize total surplus, S . The social planner's welfare maximization problem is

$$\text{Max}_{\{r_i, q_i\}} S = \int_{z=0}^Q P(z) dz - \sum_{i=1}^N c_i(r_i) q_i - w_i q_i - D\left(\sum_{i=1}^N r_i q_i\right) \quad (1)$$

The first order conditions for an interior maximum are

$$-c'_i(r_i^*) = D'\left(\sum_{i=1}^N r_i^* q_i^*\right) \forall i \in N \quad (2)$$

with q_i and r_i greater than zero and

$$P(Q^*) = c_i(r_i^*) + w_i + r_i^* D'\left(\sum_{i=1}^N r_i^* q_i^*\right), \forall i \in N \quad (3)$$

These conditions require that each firm's operations be optimized on two margins. The *efficient abatement* condition (2) ensures that abatement is both cost-minimizing (since the marginal abatement cost is equated across firms) and surplus-maximizing (since MAC equals marginal

damage). Let $MAC^* = D'\left(\sum_{i=1}^N r_i^* q_i^*\right)$ denote the common value of the $-c'_i(r_i^*)$. The *efficient*

output condition (3) ensures that output is surplus maximizing because each firm's marginal social cost equals marginal willingness to pay. Note that although condition (2) determines a unique emission rate for each firm, condition (3) determines only the aggregate level of output. Any combination of q_i^* such that the q_i^* sum to Q^* and the $r_i^* q_i^*$ sum to E^* is a solution to the surplus maximization problem.

Figure 1 illustrates the social optimum. The marginal abatement cost curve, MAC, is obtained by inverting the expression $MAC = -c'_i(r_i)$ to obtain emission rates as a function of MAC,

$r_i = r_i(MAC)$, computing aggregate emissions over all firms, $E = \sum_{i=1}^N r_i q_i^*$, and reinverting to

obtain MAC as a function of total emissions. Optimal emissions and optimal marginal abatement cost are determined in Panel (a) by the intersection of the marginal damage curve, MD, and the aggregate marginal abatement cost curve, MAC. Optimal output, Q^* , and product price, p^* , are determined in Panel (b) by the intersection of the product demand curve, D, and the long run unit social cost curve, $LAC_a = c_i(r_i^*) + w_i + r_i^* MAC^*$. The curves in the two panels are interdependent.

The position of MAC is conditional on the optimal outputs q_i^* and the position of the LAC curve is conditional on the optimal emission rates r_i^* , as determined by MAC^* .

Insert Figure 1 here

Cap-and-Trade

The social optimum can be supported as a competitive equilibrium under cap-and-trade regulation. The regulator distributes allowances A_i to each firm so that the sum of allowances granted equals the optimal level of emissions, that is, $\sum_{i=1}^N A_i = E^*$. Let P_a denote the price of allowances. Then firm i 's profit maximization problem is

$$\text{Max}_{\{r_i^a, q_i\}} \pi_i^a = P(Q)q_i - c_i(r_i)q_i - w_i q_i - P_a(r_i q_i - A_i). \quad (4)$$

The two first order conditions are

$$-c_i'(r_i^a) = P_a \quad (5)$$

if q_i is greater than zero and

$$P(Q^a) = c(r_i^a) + w_i + r_i^a P_a. \quad (6)$$

Equation (5) ensures cost-minimizing abatement and defines each r_i^a . Equation (6) requires that each firm earn zero profit, and identifies Q^a . Because the system (5) and (6) can be obtained from (1) and (2) by replacing $D'(\sum_{i=1}^N r_i^* q_i^*)$ by P_a and r_i^* by r_i^a and solution to the surplus maximization problem is a competitive equilibrium and vice versa.

The cap-and-trade equilibrium is illustrated by a simple reinterpretation of Figure 1. The MAC becomes the demand for emission allowances. The supply curve of allowances is vertical at E^* .

The price of allowances is $P_a = MAC^* = D'(\sum_{i=1}^N r_i^* q_i^*)$. The long run equilibrium output Q^* is determined by the intersection of the demand curve with the long run supply curve, which is horizontal at LAC_a .

Baseline-and-Credit

Under a baseline-and-credit plan the regulator sets an industry-wide performance standard, r^s . Firm i 's net demand for credits is $(r_i - r^s)q_i$. Negative values denote a supply of credits. If the price of credits is P_c , then firm i 's profit maximization problem is

$$\text{Max}_{\{r_i, q_i\}} \pi_i^c = P(Q)q_i - c_i(r_i)q_i - w_i q_i - P_c q_i (r_i - r^s). \quad (7)$$

The first order conditions for an interior maximum are

$$-c'_i(r_i^c) = P_c \quad (8)$$

and

$$P(Q^c) = c_i(r_i^c) + w_i + r_i^c P_c - r^s P_c. \quad (9)$$

Equation (8) is the usual efficient abatement condition which defines each r_i^c . Equation (9) is the usual zero-profit condition which determines Q^c . Let us assume that the regulator sets the emission rate standard equal to the average emission rate under the social planner scenario,

$r^s = \frac{\sum_{i=1}^N r_i^* q_i^*}{Q^*}$. If the emission standard is binding and net demand for credits equals zero, then

$\sum_{i=1}^n r_i^c q_i^c = \sum_{i=1}^n r^s q_i^c$. Substituting for r^s we can calculate that $\sum_{i=1}^n r_i^c \frac{q_i^c}{Q^c} = \sum_{i=1}^n r_i^* \frac{q_i^*}{Q^*}$. This

last equation implies that if market shares are the same under baseline-and-credit and cap-and-trade plans, then any set of emission rates satisfying the socially optimal abatement condition (2) also satisfies the corresponding baseline-and-credit equilibrium condition (8).

The zero-profit condition for baseline-and-credit equilibrium (Equation (9)) is similar to optimal equation (3) with P_c playing the role of marginal damage, D' . If emission rates are the same under the two cases ($r_i^c = r_i^*$) then $P_c = D'(E^*)$ and the right hand side of (9) is the same as the right hand side of (3) except for the term $-r^s P_c$. This shows that the output price under baseline-and-credit trading will be less than optimal. Consequently aggregate output Q^c will be larger than aggregate output Q^* chosen by the social planner.

We summarize the argument in the following propositions.

Proposition 1. Long run competitive equilibrium emissions and output are socially optimal under a cap-and-trade plan, provided the supply of allowances equals the socially optimal quantity of emissions.

Proposition 2. In long run competitive equilibrium, aggregate emissions and aggregate output under a baseline-and-credit plan are higher than the long run equilibrium levels of a cap-and-trade plan with the same aggregate emission rate.

The Experimental Environment

We wish to investigate whether Propositions One and Two will hold in a laboratory trading environment which embodies greater institutional detail than the simple competitive theory of the last section. In particular we wished to investigate whether long-run equilibrium results would be obtained when both output and emission rights (allowances or credits) were traded in a fully specified institutional environment. Because Propositions One and Two deal with long-run, zero profit equilibrium we also wished to capture the distinction between short-run production decisions and long-run capital investment decisions. Finally we wished to create an experimental environment in which the operation of alternative emissions trading plans could be demonstrated to students and policy-makers. This caused us to reject the “context-free” framing of the typical laboratory experiment in favour of a context-laden framing in which subjects were told explicitly that they were trading emission rights and making output decisions.

We settled on a frame in which subjects would be told that they represented firms producing an output at a constant variable cost up to a fixed capacity level, k . Production of the output would generate emissions at a rate of r emission-units per unit of output, q . Total fixed cost, $c(r)k$, would depend on the amount of capacity and the emission rate chosen. Emissions would be subject to cap-and-trade or baseline-and-credit regulation. Under the former subjects would receive a periodic allotment of allowances. Under the latter subjects would be assigned a common performance standard, r^s . Allowances and credits would trade in a call market, that is a sealed-bid auction in which bids and asks are ordered in descending and ascending order respectively, a market clearing price is determined, and all successful orders are traded at the market clearing price. Output units would trade in a similar call market, except that the demand side would be represented by a simulated demand curve. Financial results for each trading period would be reported in a conventional accounting framework. Capacity would have a fixed lifetime of a specified number of periods, after which it must be replaced. Subjects could adjust the amount of capacity at replacement time.

Our theoretical framework uses a simultaneous decision model where firms are expected to simultaneously choose an output and emission rate that maximizes their profit in continuous time. Our original idea was to mimic this by having subjects choose a capacity level *and* an emission rate at capacity replacement time, and to hold emission rates constant over the life of the capital stock. This would reflect the idea that emission control is built into process design and that it can

only be changed by major reinvestment. Subjects would then follow the decision pattern outlined as Algorithm 1

Algorithm 1 (simultaneous markets for emission rights and output)

```

repeat
  choose capacity (k) and emission rate (r)
  for each period in lifetime of capacity
    submit bids or asks to emissions right market
    submit asks to output market
    wait until output and emissions rights markets clear
    produce number of units sold in the output market
    redeem emissions rights, banking excess
until end of experiment

```

We quickly concluded that this algorithm would be difficult to implement in a testbedding environment. There were at least two difficulties. First, firms' ability to produce output is constrained by the quantity of emission rights which it holds, but this amount depends on the result of the emissions right market, which is unknown at the time output asks are submitted. To avoid default subjects would have to hold a large inventory of rights or else have some means of obtaining rights in a reconciliation market. More importantly, fixing both capacity and emission rate renders the short-run demand curve for permits perfectly inelastic over the interval $0 \leq p_a \leq (P - c_i(r_i) - w)/r_i$. Small variations in capacity will then lead to rapid oscillations in allowance or credit prices and consequent instability in the output market. To remedy these problems, we chose to operate the emission rights and output markets sequentially. We also allowed emission rate to vary in every period even though capacity still has a fixed, multi-period lifetime. It is as if emissions could be controlled by short-lived capital investments in end-of-pipe treatment.

Even with these modifications, it turns out that care must be taken in sequencing the events within each period. For example, we tested a version of the model in which subjects sequentially chose their emission rates, bought and sold emissions rights, and then offered contracts in the output market. This sequence is captured in Algorithm 2.

Algorithm 2 (Sequential Markets following emission rate choice)

```

repeat
  choose capacity (k)
  for each period in lifetime of capacity
    choose emission rate

```

```

submit bids or asks to emissions right market
wait until the emissions rights market clears
submit asks to output market
wait until output market clears
produce number of units sold in the output market
redeem emissions rights, banking excess
until end of experiment

```

Early simulations with robot traders quickly revealed that this sequence led to the same kind of unstable results we had feared from Algorithm 1 because capacity and emission rates were already fixed when subjects were constructing bids and asks for the emission rights market. Consequently we decided to reorder the sequence so that emission rates were set after the permit market was finished, as shown in Algorithm 3. Figure 2 shows in more detail how the algorithm was implemented.

Algorithm 3 (Sequential Markets, emission market precedes emission rate choice)

```

repeat
  choose capacity (k)
  for each period in lifetime of capacity
    submit bids or asks to emissions right market
    wait until the emissions rights market clears
    choose emission rate
    submit asks to output market
    wait until output market clears
    produce number of units sold in the output market
    redeem emissions rights, banking excess
until end of experiment

```

To guide subjects through this relatively complex series of decisions and events, we programmed an elaborate user interface. The screen shots in Appendix C give the flavour of the interface. Figure C1 shows the Allowance Order and Dataview Forms. On the right hand side of the window, the Dataview Form allows the subject to view tables recording capacity, permits, market data, balance sheets and income/expenditure statements. The planner tab, shown in the figure, reports details on the subject's capacity and emission rate in the previous period, as well as results from the output market and cash holdings. The planner tab also contains a Wizard which can be used to compute the various components of cost at different levels of output and emission rates. On the left hand side, the Allowance Order Form allows subjects to enter bids or asks for

emission rights. Up to three Bids and three Asks can be specified, each for a different price.

Figure C2 shows the Output Order Form and the Income Statement tab of the Dataview Form. The Output Order Form is similar to the Allowance Order Form, except that only asks are permitted. The income statement shows the financial results reported at the end of a period. Revenues from sales of output and emission rights are booked at transaction value. Revenues also include the implied value of allowances received. These have been booked at the market price in the previous period. The cost of goods sold includes materials costs, the cost of emission rights used in production (at book value) and the book value of emission rights sold. Depreciation on fixed capital and Net Income for the period is also reported.

Figure C3 shows the Capacity Order Form and the Ledger tab in the Dataview Form. The ledger tab shows all the bookkeeping entries that have been generated by the subject. For example, lines three and four show the effect of building 10 units of capacity at \$102.593 each. The total amount, \$1025.93, as been debited to Fixed Plant and credited to Cash.

Parameterization

The experimental environment contains parameters specifying the slope and intercept of a linear demand curve, the shape of the unit capacity cost curve, the level of operating costs, allowance endowments and performance standards, initial holdings of cash and emission rights, “results” from the period preceding the start of the experiment and, of course, the number of firms in the market. For the simulated sessions reported below we chose an inverse demand curve with equation $P=100-Q$.

The unit capacity cost function is $c_i(r_i)=u_0+(u_1-u_0)((r_{\max}-r_i)/r_{\max})^{\alpha_i}$. Parameters u_1 and u_0 specify maximum and minimum unit costs, respectively, and α_i determines the curvature of the unit MAC curve. To simplify the choices facing subjects, emission rates were restricted to integer values in the range $[0, \dots, r_{\max}]$ with maximum emission rate $r_{\max}=3$.

This implies that the smooth MAC curve of Figure 1 is replaced by a step function with steps equal to the difference in costs $c(r_i)-c(r_i-1)$ at successive integer values.

We specified a population of eight firms, two of each of four cost types. Each firm has the same cost structure, but the parameters of the function itself are different for the four firm types. A spectrum of firm types rated on a scale from A, using the cleanest technology, to D using the dirtiest technology, is used. The dirty firms have lower uncontrolled unit costs compared to firms with the cleaner technology, $c_{\text{dirty}}(r_{\max}) < c_{\text{clean}}(r_{\max})$, but it is cheaper for the clean technology firms to abate pollution than it is for the dirty firms. This results in dirty firms having higher units costs than the cleaner ones at the lowest emission rates, $c_{\text{dirty}}(0) > c_{\text{clean}}(0)$. To keep things simple the unit variable cost was assumed to be zero, $w_i=0$ for all i . Table 2 shows the parameter values for each of the four firm types. For cap-and-trade treatments allowances equal to the optimal emissions

were distributed between the firms, and for baseline-and-credit treatments the emission rate standard was chosen to be the average emission rate implied by the optimal amount of emissions and optimal quantity of output. Table 3 shows the theoretical predictions of the model using these parameters.

Tables 2 and 3 about here

Robot Traders

We tested our software by creating robot strategies to make the decisions required of human subjects. Although the primary focus of this paper is not to create artificial intelligence traders that operate exactly the way humans would, we did want to embody decision rules that were both simple and reasonable. The purpose of the robots were to test the logic of the software, to illustrate the interactions between the markets, and lastly to primitively simulate results of experiments. Throughout, we assume that our robots are price-takers that use profit maximization principles when making decisions and whose expected value of a variable is simply its value from the current or last period. Details are reported in the following paragraphs.

The first event in a typical period is the conducting of trade in the call market for allowances or credits. This takes place under a fixed capacity for producing output, but before an emission rate has been chosen for the period. At this juncture, since emission rate is unknown, a bid and ask strategy is constructed that reveals the robot's entire demand and supply schedule for emission rights, allowing for any possible subsequent choice of emission rate. The exact bids and asks depend on the firm's established capacity and its inventory of allowances or credits. The robot assumes that output will be at capacity because it takes output price as given and it has constant marginal cost. The firm will make a bid or an ask for each possible selection of r . The robot considers each emission rate, from 0 to 3 in succession, deciding whether it will need to buy emission rights to reach this goal or whether it will have rights left over to sell off. If allowances or credits need to be bought, the robot submits a bid for the needed amount, net of any previous amounts needed to achieve previously considered emission rates, at a price equal to the unit cost increase of lowering its emission rate by one unit. If the robot has allowances or credits to sell at the considered emission rate, then the extra ones beyond what would be needed at the previous emission rate, are priced at the cost savings of raising the emission rate by one unit. The bids and asks generated from this algorithm are profit maximizing in the way that any price outcome in the market results in the robot making a profit maximizing trade at an implied profit maximizing emission rate.

Before considering the next phase, let us investigate how these decisions differ between our two emission trading institutions. Under a cap-and-trade scheme the above robot rationality implies that allowances are priced at their use value, their cost savings amounts during the current period. Under a baseline-and-credit regime there is a unique problem. Using this rule, under the credit system the first credits are priced at their cost, the cost increase of reducing emission rate one unit

below the emission rate standard. However, these credits were produced at the end of last period. One might argue that the cost of creating the credits during the preceding period is sunk, and that they have a value this period equal to the cost decrease of being able to increase the emission rate one unit above the standard. With a continuous emission rate and cost function these two are identical, but they are not identical when using discrete emission rates and cost functions. We assume our robots with the original rationality in this case. The issue is similar to the general distinction between production in advance and production to demand. When comparing these two models of production, do we expect agents to price inventory at marginal cost under advance production assumptions, or do we expect agents to treat the production as a sunk cost and price their inventory at zero? Evidence from experimental advance production markets points to advance production inventory being priced at marginal cost. This assumption is a good fit with our environment as credits would not be created in the first place unless subjects valued them enough to incur the required higher unit capacity cost.

In the next phase each subject must choose an emission rate. Since our robot is a profit maximizer, our robot simply chooses the discrete value of r between 0 and 3 which yields the highest expected profit. In choosing so, the robot simply chooses r such that the MAC equals the price of allowances or credits from the current period. The robot does face a few restrictions. First, total capacity cost at the chosen emission rate, $c_i(r_i)k$, must be affordable. Secondly, if the robot did not sell its entire capacity last period then the robot will definitely not choose to increase emission rate from last period, due to the possibility of a deficit in available permits.

Next, the output market uses a call market mechanism to elicit a supply curve from subjects. The robot traders are programmed to price their output at total marginal variable cost, since the unit capacity cost, $c_i(r_i)k$, is sunk at this point. The notion of a sunk cost is now considered by the robot because capacity and emission rate are already set and since output inventory cannot be carried over from period to period in the way that credits can. Therefore, the output is priced at the variable cost w_i , equal to zero in our simulations, plus the value of the used allowances or credits evaluated at their cost-basis. The firm's cost-basis is the average value of the allowances or credits that it has entered in inventory. Each time an allowance or credit is bought its price augments the cost-basis. When emission rights are used, or sold, they are removed from inventory at the current cost-basis value. At this price the robot sells its entire capacity, unless it does not have enough emission rights to do so. If the robot is facing a baseline-and-credit plan, and if the robot's emission rate is lower than the emission rate standard, then credits are being created. These created credits are evaluated at the current period's market price, and act as a subsidy entering as a negative term in the price of the output offer.

Lastly, is the capacity choice decision. To allow for a simplistic design which did not require robots to consider whether other robots were changing capacity at the same time, it was decided that each of the eight firms would choose capacity in separate periods. To allow for a slow and clean convergence to an equilibrium, we restrained the robots from raising or lower their

capacities by more than one unit at a time. Because the robots are price takers with constant marginal costs, profit maximizing robots which are earning positive marginal profit this period and currently selling at capacity will raise their capacity by one unit. If robots are earning negative profits this period, or if they did not sell their entire capacity during the current period, then they will lower their capacity by one unit. If none of these conditions are met then the robot will not change its capacity. To allow for rounding errors positive profit is defined to be greater than \$1 and negative profit is defined to be less than -\$1. In a baseline-and-credit equilibrium total and marginal profits will be zero, due to constant price and constant marginal cost assumptions. However, firms in a cap-and-trade equilibrium could earn positive total profits and zero marginal profits because of the fixed endowment of allowances given to the firm at the start of the period.

Simulated Sessions

In this section we report a simulated experiment designed primarily to test the operation of the software. In addition to demonstrating the feasibility of the computerized environment, we wanted to investigate whether a change in rules from cap-and-trade to baseline-and-credit trading would lead to the higher levels of output and emissions predicted by Proposition 1 and whether the stability of the system would be affected by random errors in decision-making by the subjects. We compared two institutional conditions: a switch to cap-and-trade rules starting from the predicted equilibrium under baseline-and-credit trading and vice versa. We considered three levels of decision error: none, low and high. In the no decision error treatment all robots followed deterministic strategies as described above. In the remaining two treatments robots submitted bids and asks chosen from a normal distribution around the profit maximizing price. The small decision error treatments chose prices from a normal distribution with a standard deviation set equal to 5% of the profit maximizing price, therefore with a coefficient of variation of 5%. The large decision error treatment was constructed assuming a 15% coefficient of variation. Table 4 illustrates this 2x3 factorial design. We report the results from only one replication of each cell. The observations below are based on simulation results illustrated in Figures 3 through 8.

Table 4 about here

Figure 3 shows a simulated baseline-and-credit plan, with no decision error, starting from cap-and-trade equilibrium values. We see at the start of the period that credit price in the bottom-left panel is high because firms are not initially provided with any credits to sell or use. Subsequent emission rate choices are all low because no firm can procure the credits needed to emit above the emission standard. As credits become available in the succeeding periods they feed output expansion as the emission rate standard acts as an output subsidy. Consequently, emission rates return to their equilibrium, and capacity, output and aggregate emissions climb smoothly to their new, higher equilibrium levels. Notice that capacity overshoots its equilibrium value as the sequence of capacity expansion getting to the equilibrium level leads to an unstable distribution of output between firms that must unravel itself appropriately until aggregate capacity drops down

to the equilibrium output level.

Figure 4 illustrates the cap-and-trade simulation using robots making no decision errors and starting parameters at their credit equilibrium values. The top-left panel shows how capacity and output quickly and smoothly converge to their equilibrium level. Once they have done so however, rearrangements toward an equilibrium distribution of output between the firms, causes minor perturbations in the allowance market in the bottom-left panel. This is caused by firms changing capacities, and hence switching between the role of buyer and seller, due to the fact that allowance distribution between the firms is constant. These very minor deviations in the credit market result in subsequent discrete emission rate changes in the bottom-right panel. These emission rate fluctuations cause aggregate emissions to follow an oscillating convergence pattern toward equilibrium.

Figures 5 through 8 show that when robots make errors in their pricing decisions the results are very different. Generally speaking, while under the no decision error assumption robot simulations show more volatility on the convergence path under a cap-and-trade plan than under a baseline-and-credit plan, this does not hold true when robots make decision errors. The baseline-and-credit sessions in Figures 5 and 7 show more volatility in the output market, the emission rights market, emission rate choice, and aggregate emissions than the corresponding cap-and-trade simulations in Figures 6 and 8. The discrepancy is most pronounced when comparing the top-right and lower-left panels of Figures 7 and 8 involving large decision errors. The wild oscillations in aggregate emissions under the baseline-and-credit plan could have very large welfare repercussions due to the environmental damage function being weakly convex. Baseline-and-credit institutions cause more volatility than cap-and-trade institutions when firms make errors because they can fuel their own mistakes, emission-wise, due to their variable emission baseline. Random decision error is not as much of a problem under a cap-and-trade system, due to the fixed allocation of allowances every period.

These findings may be summarized in the following observations.

Observation 1: With the assumption of no decision error, robot traders under both cap-and-trade and baseline-and-credit emission trading schemes converge to their respective predicted equilibria.

Observation 2: Each emission trading institution's output market convergence pattern seems to frequently under-supply output when decision errors are made, but both treatments' capacity converges to its equilibrium level from above, whether there are small, large, or no decision errors at all.

Observation 3: While the call auction trading mechanism seems robust in the face of robots making decision errors, the variable baseline and credit lag inherent to the baseline-and-credit emission trading scheme makes the credit market more volatile than the allowance market.

Observation 4: Decision errors greatly increase the volatility of aggregate emissions under a baseline-and-credit plan while only mildly raising the volatility of emission under a cap-and-trade plan.

Conclusions

This paper is the first report from an ongoing research. To date, we have focussed on designing and implementing a computerized laboratory environment suitable for testing long run predictions about alternative emission trading plans. Substantial progress has been made. We have a working program which permits emission rates and output to be determined in interrelated markets. Still, much work remains before the experiment can produce credible results from human subjects. At the technical level, we have discovered that robot strategies will achieve equilibrium only when the parameters are carefully chosen to avoid a large range of equilibrium prices. We believe this may be due to the small number of discrete steps permitted in emission rate choice and we plan to expand the number of steps to mitigate this problem. The current robot strategies must also be constrained to adjust capacity one unit at a time on a rotating basis. This leads to slow convergence to equilibrium and artificially long capital lives. We will investigate whether this constraint could be relaxed if capacity decisions were explicitly related to profit levels.

A second area for development is at the human level. The decision making environment is complex and we must investigate whether it can be effectively communicated to human subjects. Pilot sessions using the current interface are planned for this summer. After revisions to accommodate what is learned from these sessions we hope to progress rapidly to testing the basic propositions with human subjects.

References

Tables

Table 1. Notation

Variable/Function	Description	Properties
W	Total Welfare	$W = W(Q, E)$
Q	Aggregate output	$Q \geq 0$. Denoted with superscripts '0', '*', 'a', and 'c' for uncontrolled, optimal, allowance and credit cases respectively. $Q = \sum_{i=1}^N q_i$
E	Total emissions	$E = r$
r	Emission rate	$r \geq 0$. Denoted with superscripts '0', '*', 'a', and 'c' for uncontrolled, optimal, allowance and credit cases respectively.
w	Unit variable cost	$w \geq 0$. Constant.
P(Q)	Inverse demand function	$P(Q) \geq 0$, $P'(Q) < 0$.
c(r)	Unit capacity cost	$c(r) \geq 0$, $c'(r) < 0$, $c''(r) > 0$.
D(rQ)	External damage function	$D(rq) \geq 0$, $D'(rQ) > 0$, $D''(rQ) > 0$
Pa	Price of allowances	$P_a \geq 0$
Pc	Price of credits	$P_c \geq 0$

Table 2. Cost Parameters

Firm Type	u1	u0	α	w_i	Optimal Emission Rate	Allowance Endowment
A - lowest MAC	76	65	3	0	0	2
B - lower MAC	89	59	3	0	1	4
C- higher MAC	90	59	3	0	2	4
D- highest MAC	269	52	3	0	3	2

Table 3 Predictions

Trading Institution	Price of Allowances or Credits	Output Price	Aggregate Output	Aggregate Emissions	Active Firm Types
Uncontrolled	-	52	48	144	D
B&C	8	68	32	32	A,B,C,D
C&T (Optimal)	8	76	24	24	A,B,C,D

Table 4 Experimental Design: Number of Simulated Sessions by Treatment

Trading Institution	Initial Equilibrium	Error condition		
		No Errors	Low Errors (COV = 5%)	High Errors (COV = 15%)
B&C	C&T	1	1	1
C&T	B&C	1	1	1

Notes:

B&C and C&T denote Baseline-and-Credit and Cap-and-Trade, respectively

Figures

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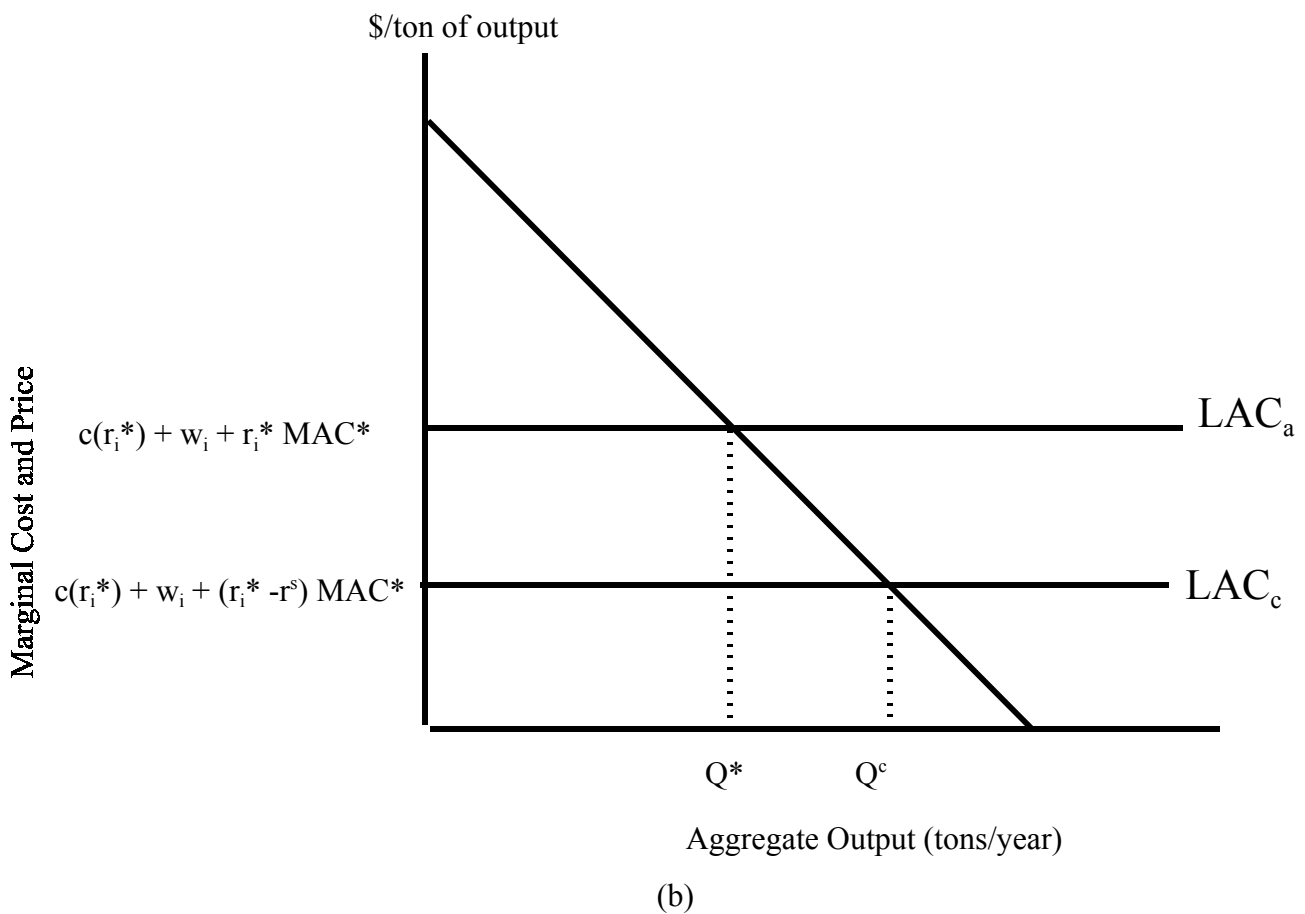
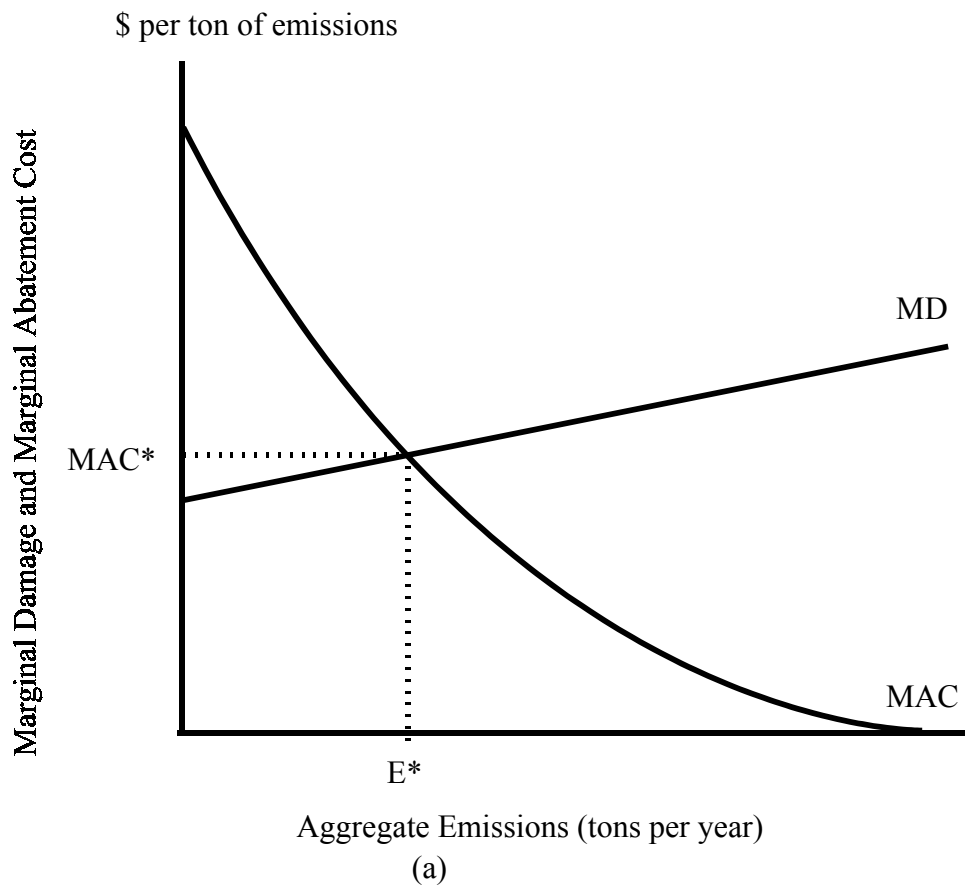


Figure 1: Surplus-Maximizing, Cap-and-Trade and Baseline-and-Credit equilibria.

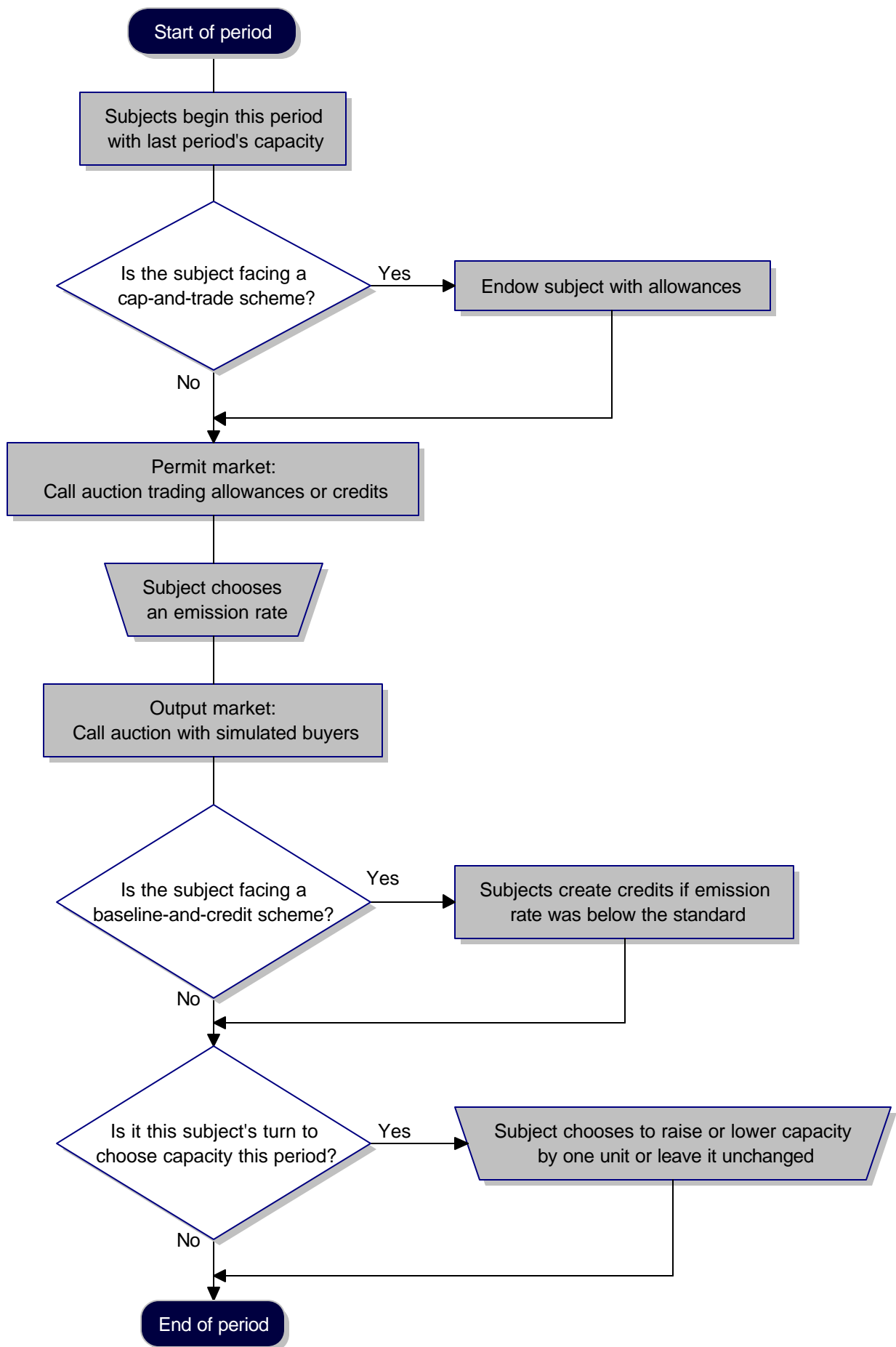


Figure 2: Sequence of events in a typical period

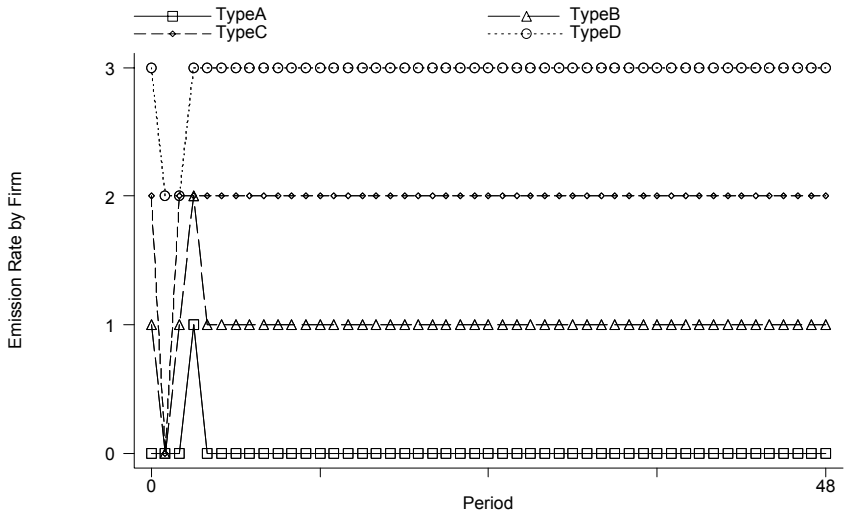
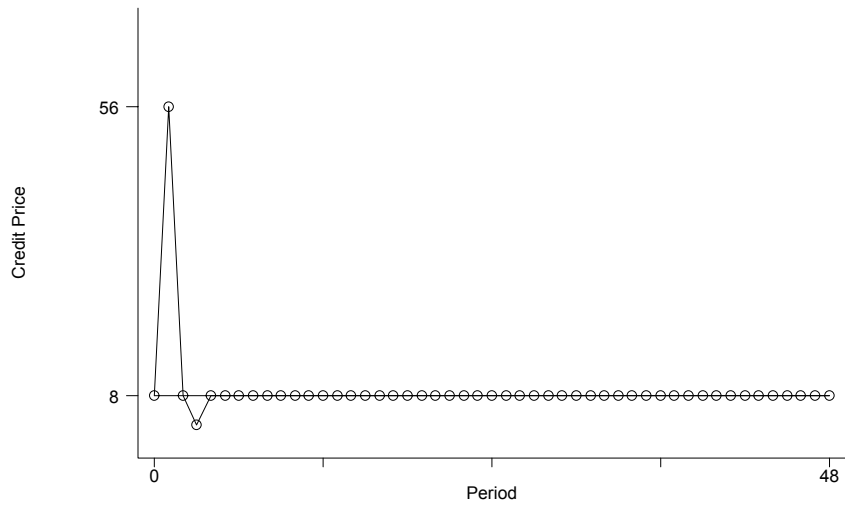
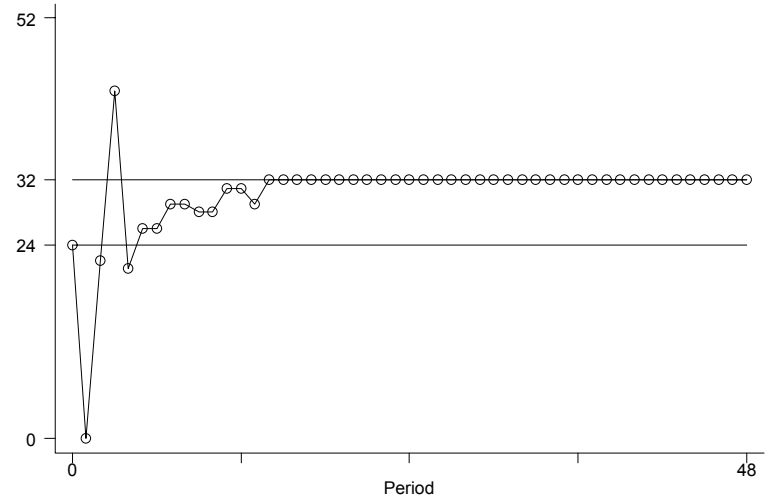
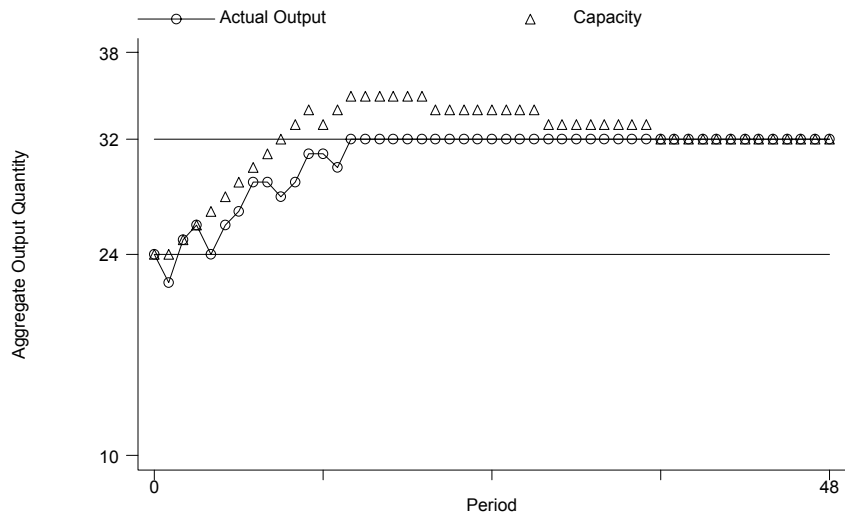


Figure 3: Allowance to credit equilibrium, no decision errors

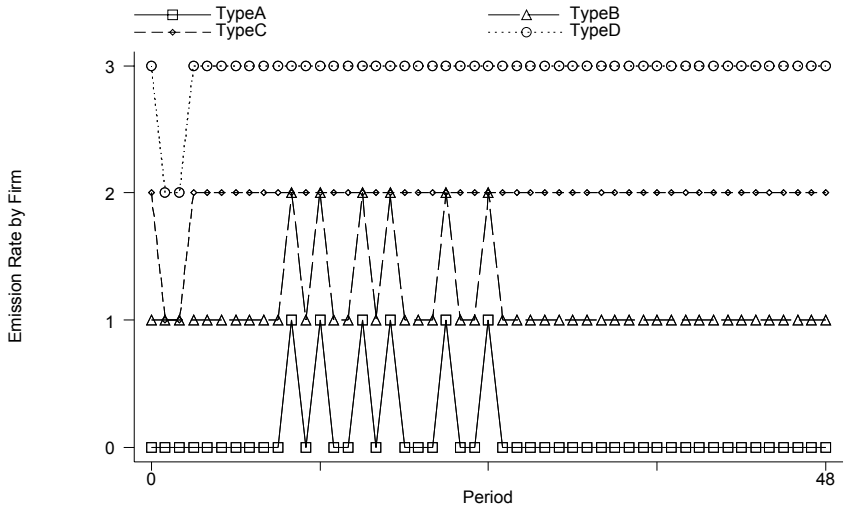
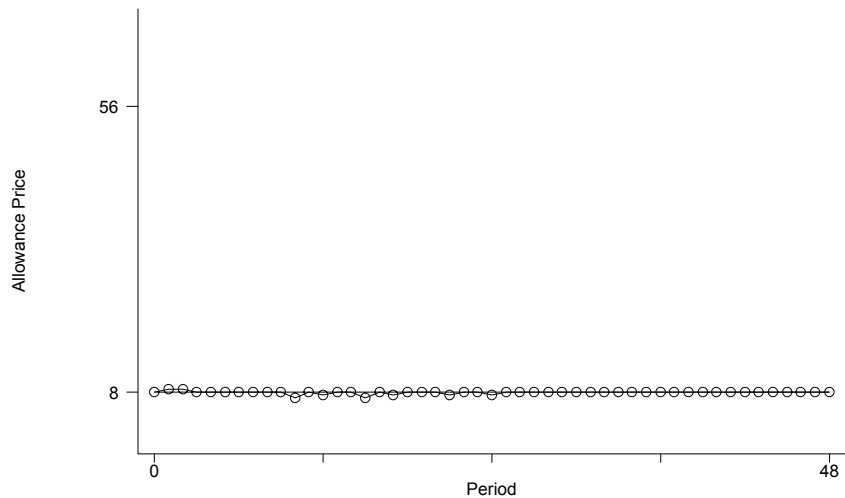
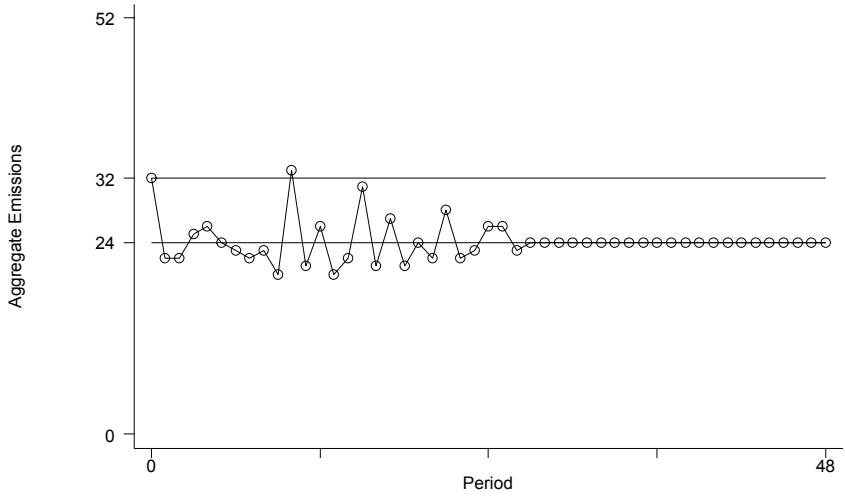
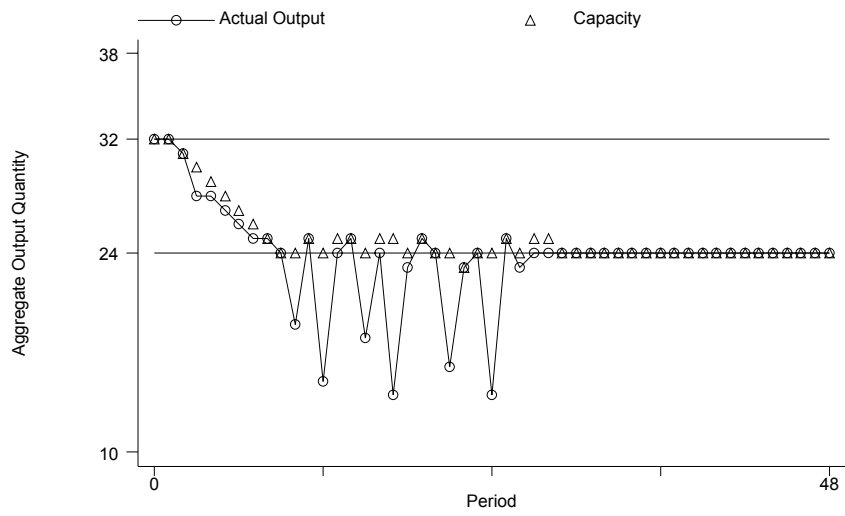


Figure 4: Credit to allowance equilibrium, no decision errors

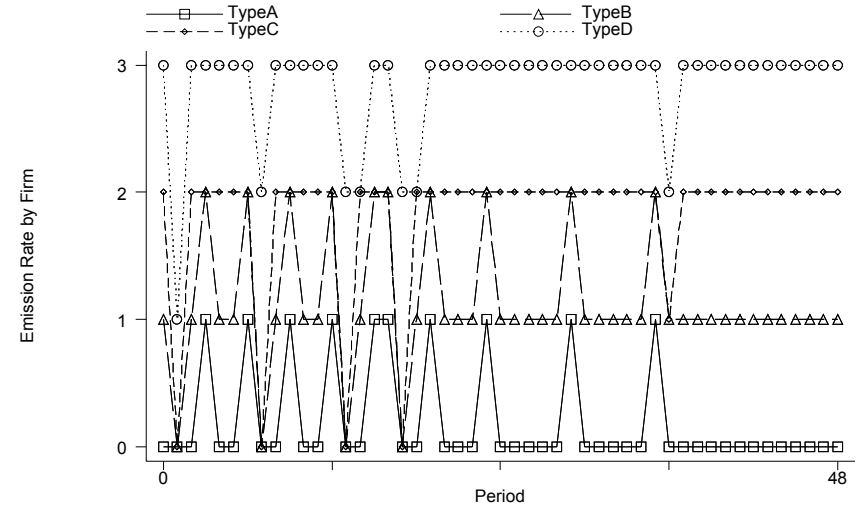
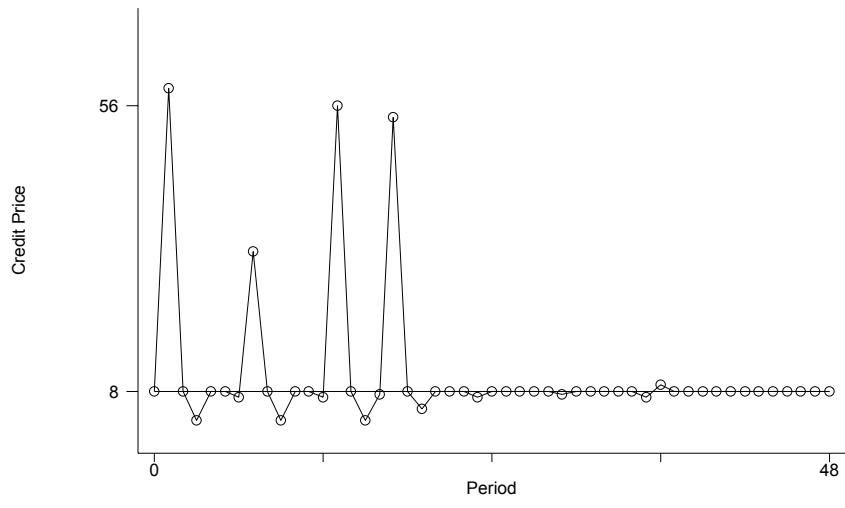
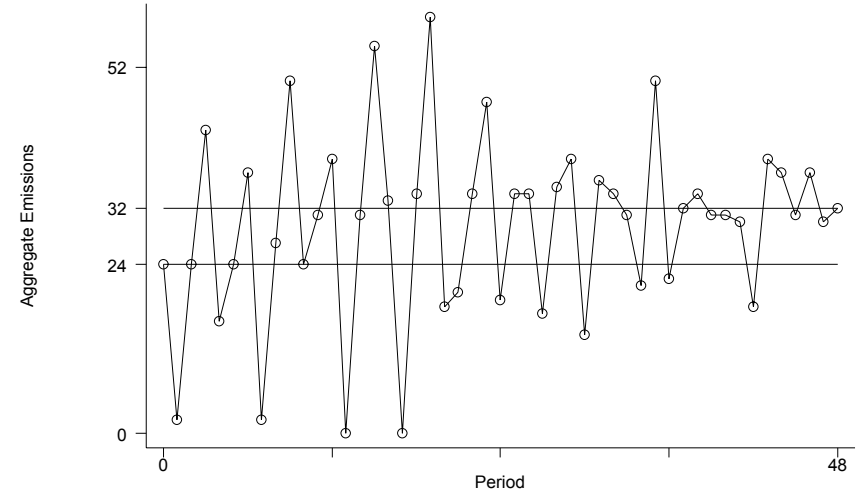
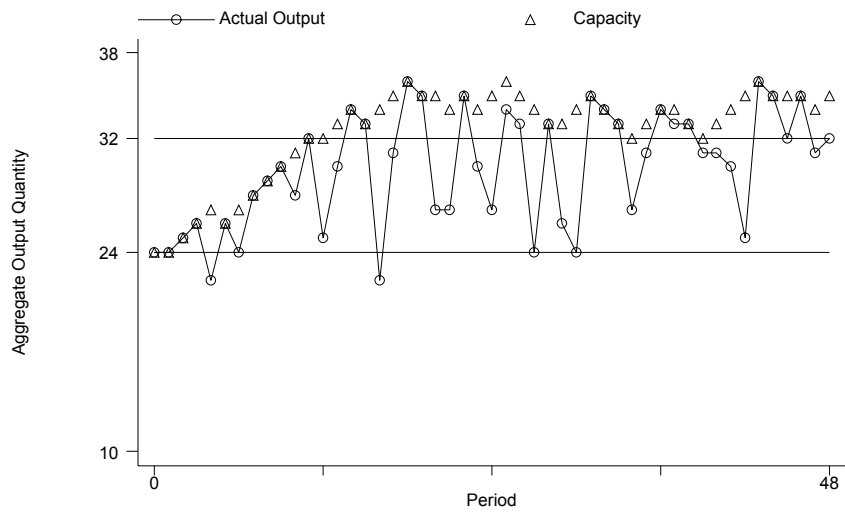


Figure 5: Allowance to credit equilibrium, small decision errors

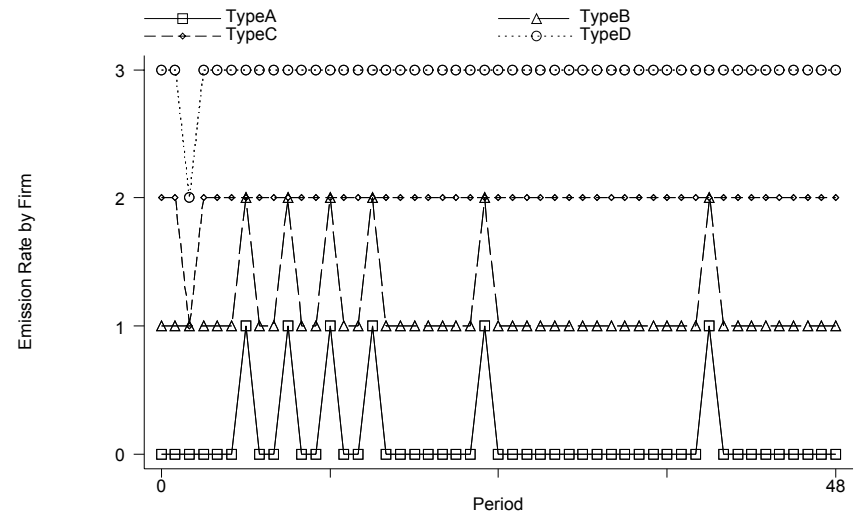
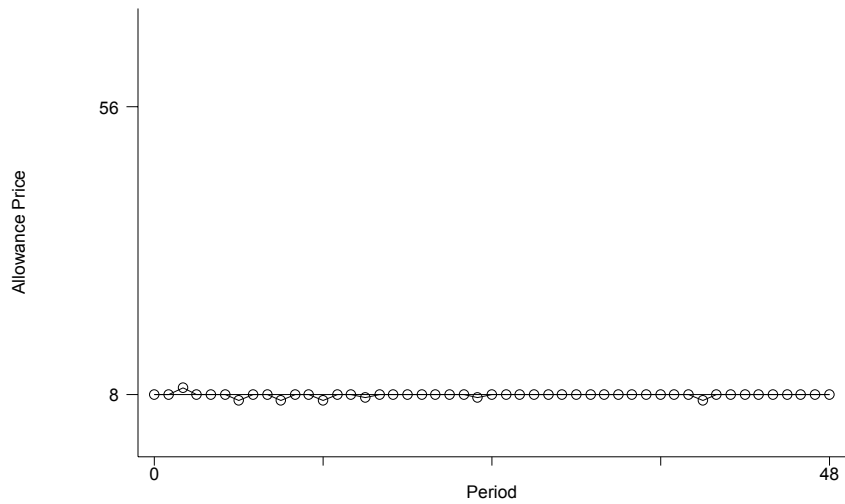
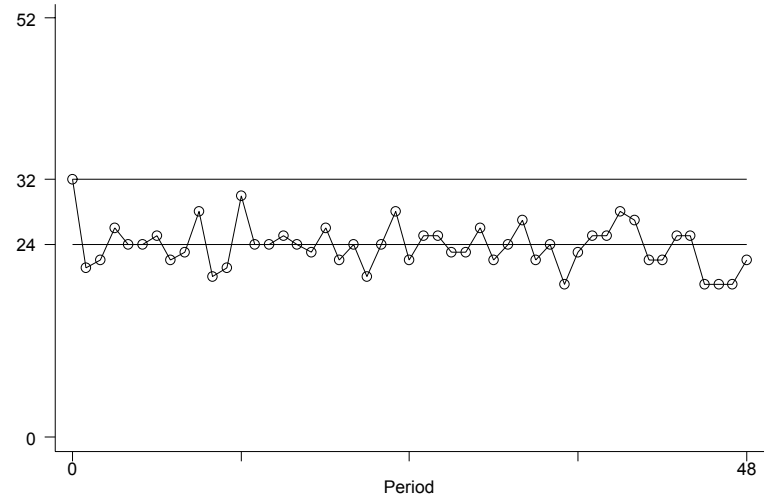
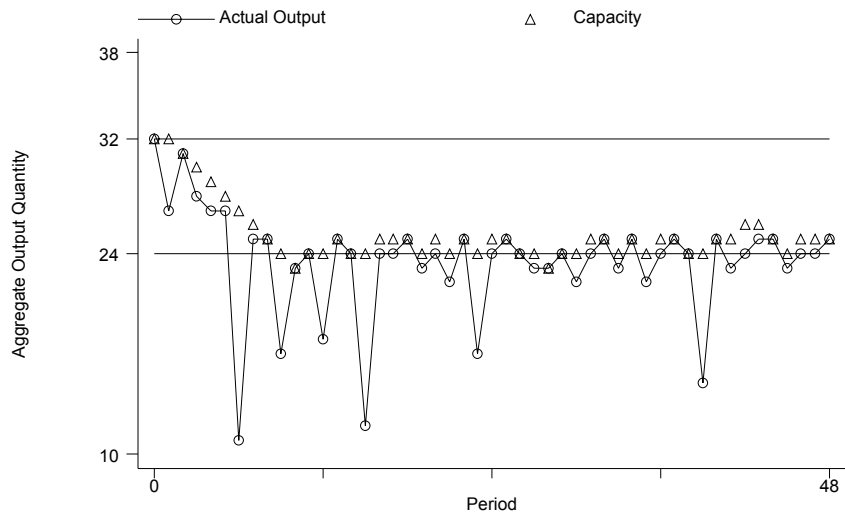


Figure 6: Credit to allowance equilibrium, small decision errors

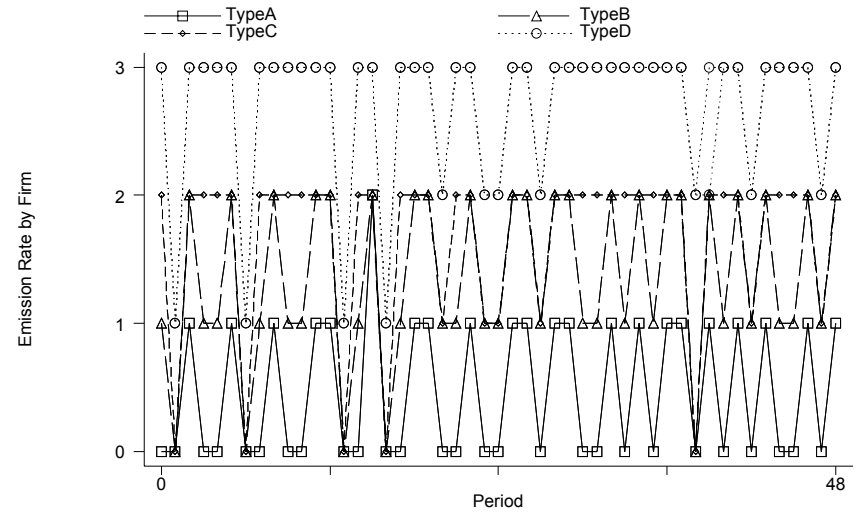
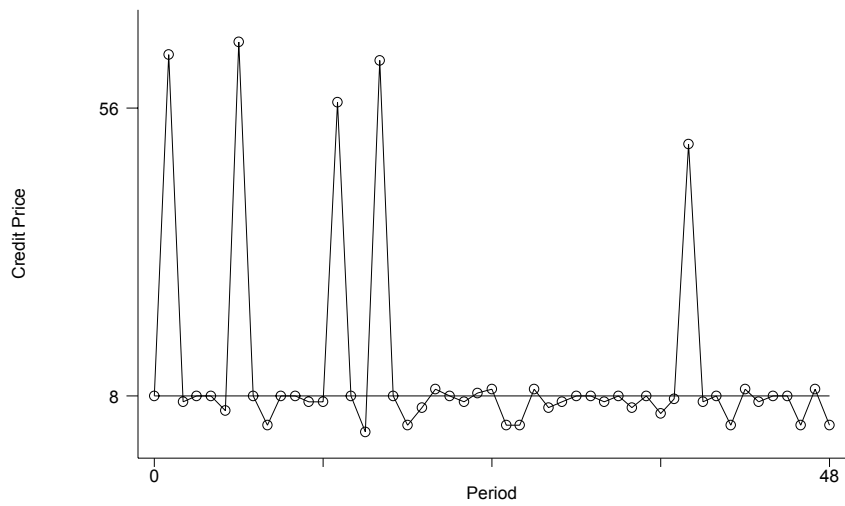
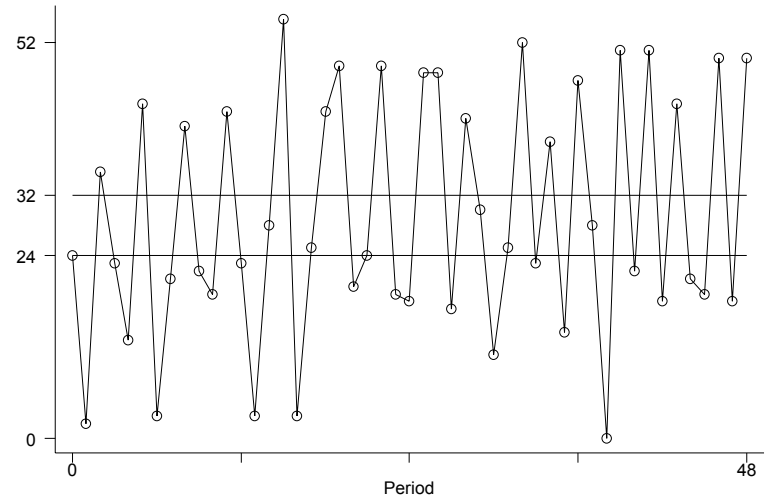
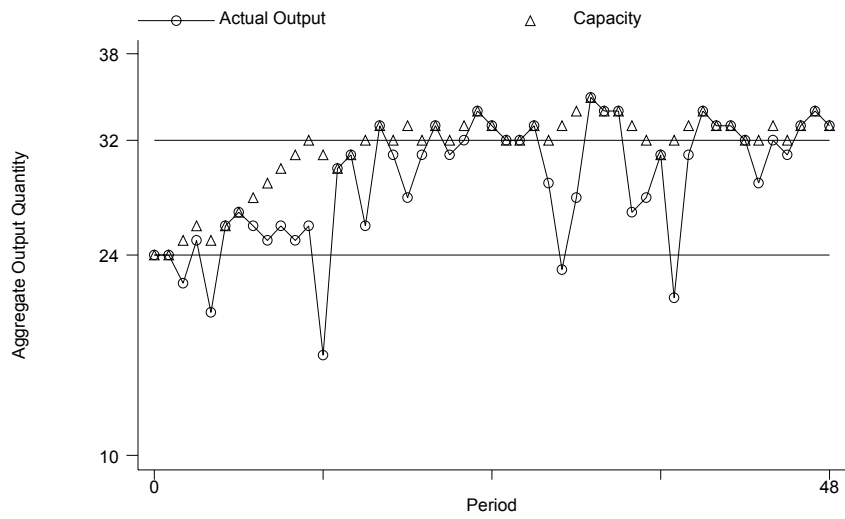


Figure 7: Allowance to credit equilibrium, large decision errors

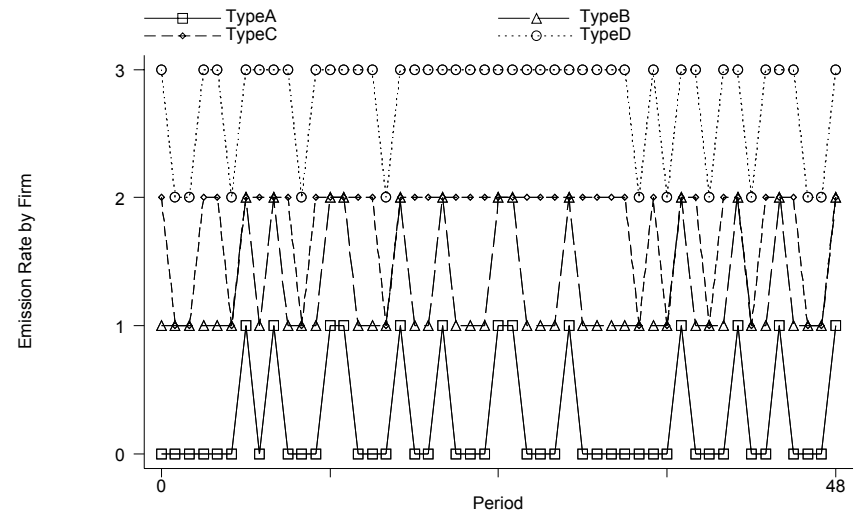
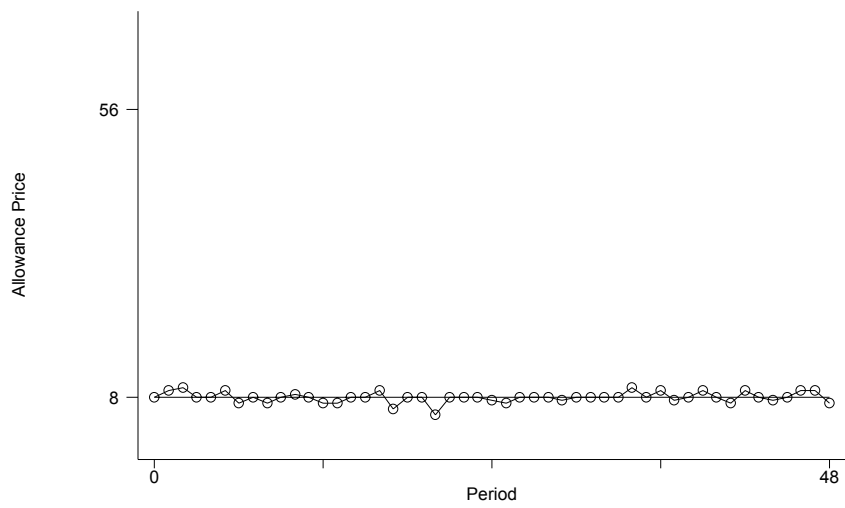
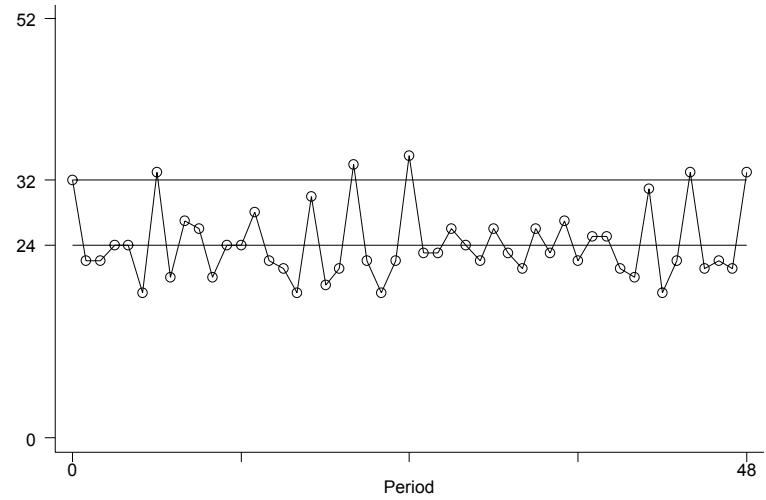
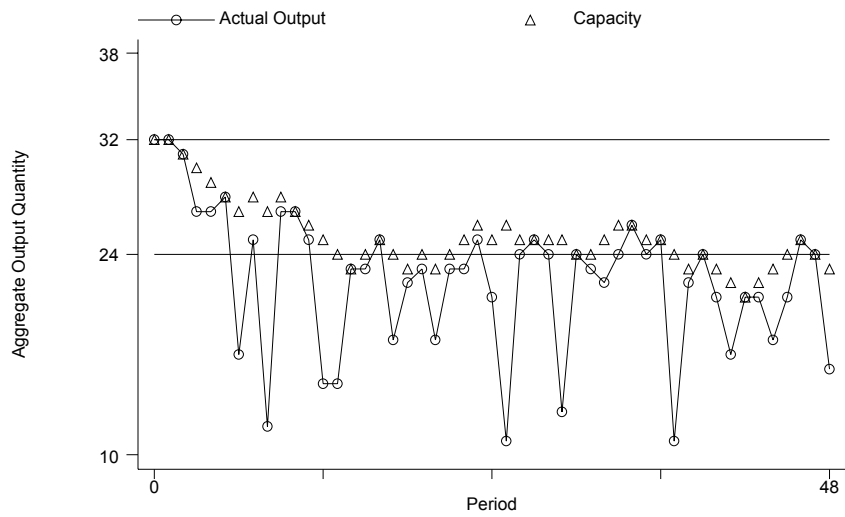


Figure 8: Credit to allowance equilibrium, large decision errors

Appendix A: A Representative Agent Model

This section uses a simple representative agent modeling technique to compute the equilibrium conditions under the different pollution abatement institutions. Underlying the entire economic model is the fact that firm's production of output creates a pollution externality, which causes environmental damage. The firm faces a downward sloping inverse demand curve for its output: $P(Q)$. The firm's costs are derived from two sources. The unit capacity cost is a cost of production infrastructure which has a constant marginal value and is a decreasing weakly convex function of the firm's emission rate, r : $c(r)$. The unit variable cost, w , is a constant variable cost embodying the value of labour and raw materials that are needed to create a unit of output, and it is not a function of the firm's emission rate. The environmental damages are assumed to be an increasing and weakly convex function of total emissions: $D(rQ)$.

Before analyzing the emission permit trading institutions, it is important to point out the optimal equilibrium conditions embodied by the traditional Pigouvian tax scenario. A social planner would want to choose emission rate and output in order to maximize total welfare, and so the social planner's welfare maximization problem is

$$\text{Max}_{\{r, Q\}} \text{Total Welfare} = \int_{z=0}^Q P(z)dz - c(r)Q - wQ - D(rQ) . \quad (\text{A1})$$

The two first order conditions are

$$-c'(r^*) = D'(r^*Q^*) \quad (\text{A2})$$

and

$$P(Q^*) = c(r^*) + w + r^*D'(r^*Q^*) , \quad (\text{A3})$$

as long as output is greater than zero. The second order conditions are detailed at the end of this section. The second order conditions of this problem are sufficient to prove that the critical values implied by the above solution are consistent with a relative maximum. These two familiar first order conditions say that the emission rate should be set so that the marginal abatement cost is equal to the marginal damage, and that the output should be set such that the marginal benefit of another unit of output equals its marginal cost, including the indirect damage cost. A Pigouvian emission tax set equal to $D'(r^*Q^*)$ will be optimal in the sense that it will elicit an emission rate of r^* and an output of Q^* from the profit maximizing representative firm. The optimal total emissions in the system will be $E^* = r^*Q^*$.

At this point it is useful to specify the equilibrium conditions for an unregulated profit maximizing representative firm for comparison. The firm's profit maximizing problem and its solution look very similar to the optimal case above only it is devoid of any reference to the external damage

function

$$\text{Max}_{\{r,Q\}} \pi^0 = P(Q)Q - c(r)Q - wQ. \quad (\text{A4})$$

The two first order conditions are

$$-c'(r^0) = 0, \quad (\text{A5})$$

as long as output is greater than zero, and

$$P(Q^0) = c(r^0) + w. \quad (\text{A6})$$

The latter is the zero profit condition and the former results from the fact that emission rates are uncontrolled and thus maximized in order to decrease the unit capacity cost which is a decreasing weakly convex function of emission rate. Equation A2 and A5 together imply that $r^0 > r^*$, and the fact that $c(r^0) < c(r^*)$ and that the marginal damage term in equation A3 is positive means that equation 6 leads to $P(Q^0) < P(Q^*)$ and thus $Q^0 > Q^*$. Total uncontrolled emissions will be greater than optimal, $E^0 = r^0 Q^0 > E^* = r^* Q^*$.

A cap-and-trade emission permit scheme with a fixed allocation of permits can be shown to achieve the optimal total emissions. Under this scheme a regulator assigns a fixed number of emission permits to each firm, which we will name 'allowances' so as to reduce confusion with permits used in other permit trading institutions. Any unused allowances can be sold, and any needed ones can be bought in an allowance market. Let P_a represent the price of allowances under cap-and-trade and 'A' be the firm's allotment of allowances. Assuming perfect competition in this industry implies that the firm does not believe its own behaviour will impact the industry price or output level and so the firm's profit maximization problem is just

$$\text{Max}_{\{r,Q\}} \pi^a = P(Q)Q - c(r)Q - wQ - P_a(rQ - A). \quad (\text{A7})$$

The two first order conditions are

$$-c'(r^a) = P_a \quad (\text{A8})$$

if output is greater than zero, and

$$P(Q^a) = c(r^a) + w + r^a P_a. \quad (A9)$$

These two conditions say that profit will be maximized when the marginal abatement cost is equal to the price of allowances and the price of output is equal to the marginal cost of output. If the regulator sets the aggregate allowance allocation equal to the optimal level of emissions, then in equilibrium $rQ = A = E^*$. Comparing the cap-and-trade first order conditions with the optimal ones illustrates that the price of allowances will be equal to the marginal damage in equilibrium, $P_a = D'(r^*Q^*)$, since equations A5 and A6, and A8 and A9 are of the same form. This also implies that the emission rate and output under the cap-and-trade scheme will be identical to those of the optimal policy: $r^a = r^*$, $Q^a = Q^*$ and $P(Q^a) = P(Q^*)$. This implies that the total emissions under the cap-and-trade scheme will be optimal, $E^a = r^a Q^a = E^* = r^* Q^*$.

A baseline-and-credit emission permit scheme with an allocation of permits proportional to output can also be used to control emissions. In this context we refer to permits as being ‘credits’ as this scheme can be thought of as an emission reduction credit system where firms face an emission standard. Emission rates above the standard require firms to buy credits and emission rates below the standard generate credits which can be sold. Let P_c represent the price of credits under a baseline-and-credit scheme and r^s be the emission standard. It is assumed that the emission standard is binding and that regulators have chosen a standard tighter than the uncontrolled rate, so $r^s < r^0$. Again, assuming that the firm is a price taker under perfect competition the representative firm’s profit maximization problem is

$$\text{Max}_{\{r, Q\}} \pi^c = P(Q)Q - c(r)Q - wQ - P_c Q(r - r^s), \quad (A10)$$

where $Q(r - r^s)$ is the firm’s net demand for credits. The two first order conditions are

$$-c'(r^c) = P_c \quad (A11)$$

if output is non-zero and

$$P(Q^c) = c(r^c) + w + r^c P_c - r^s P_c. \quad (A12)$$

Notice that the marginal revenue equals marginal cost condition now involves a new term, $-r^s P_c$, due to a subsidy inherent in the setting of an emission rate standard. Net demand for permits must be non-positive and assuming that the performance standard is binding in equilibrium we have $r^c = r^s$. This leaves equation A12 to be $P(Q^c) = c(r^c) + w$. This new formulation has the same form as

the unregulated profit maximizing condition in equation A6, only here the emission rate is lower since we are assuming $r^c = r^s < r^0$, and so $c(r^c) > c(r^0)$. This means that $P(Q^c) > P(Q^0)$ and so $Q^c < Q^0$. In order to assess how the baseline-and-credit institution compares to the optimal solution or the cap-and-trade case we must make an assumption regarding the emission standard. Suppose the regulator sets the performance standard equal to the socially optimal rate, which is also the cap-and-trade equilibrium rate, then $r^c = r^s = r^a = r^*$. First, equations A8 and A11 testify that the price of allowances in equilibrium will equal the price of credits in equilibrium, $P_c = P_a$. Comparing the marginal revenue equals marginal cost equations A9 and A12 show that when $r^c = r^a$ that price of output under the credit scheme will be lower than under the allowance scheme due to the effect of the emission standard as a subsidy on output result, $P(Q^c) < P(Q^a)$. This means that output will be greater under the baseline-and-credit scheme than under the cap-and-trade scheme or the optimal policy, $Q^c > Q^a = Q^*$. Consequently aggregate emission rates under the baseline-and-credit scheme will exceed optimal levels, $E^c = r^c Q^c > E^* = r^* Q^*$.

The solutions to all of the optimization problems above are all relative maxima. Since the proof is similar for all the above cases, we will only illustrate the proof for the socially optimal case below. To recap, the welfare maximization problem is

$$\text{Max}_{(r,Q)} \text{ Total Welfare } (W) = \int_0^Q P(z) dz - c(r)Q - wQ - D(rQ). \quad (\text{A13})$$

The two first order conditions are

$$W_r = Q^*[-c'(r^*) - D'(r^*Q^*)] = 0 \quad (\text{A14})$$

and

$$W_Q = P(Q^*) - c(r^*) - w - r^* D'(r^*Q^*) = 0. \quad (\text{A15})$$

It is obvious that the W_r condition can be satisfied if Q^* equals zero or if the bracketed term equals zero. Since we are only interested in checking whether the positive output solution is a relative welfare maximum, we assume that the top first order condition implies that $-c'(r^*) = D'(r^*Q^*)$. The necessary and sufficient second order conditions for a relative maximum at this critical point are that $W_{rr} < 0$, $W_{qq} < 0$ and $W_{rr}W_{qq} - W_{rq}^2 > 0$. In our welfare maximization context, based on our assumptions on the cost and environmental damage functions, these bivariate conditions are met. Using the fact that $r^* > 0$, $Q^* > 0$, $c'(r) < 0$, $c''(r) > 0$, $D'(rQ) > 0$, $D''(rQ) > 0$, and $P'(Q) < 0$ we can show that

$$W_{rr} = Q^*[-c''(r^*) - Q^*D''(r^*Q^*)] < 0, \quad (\text{A16})$$

and

$$W_{QQ} = P'(Q^*) - r^{*2} D''(r^*Q^*) < 0. \quad (A17)$$

For the last second order condition we must use the fact that

$$W_{rQ} = -c'(r^*) - D'(r^*Q^*) - r^*Q^*D''(r^*Q^*) = -r^*Q^*D''(r^*Q^*) \quad (A18)$$

Notice how the cross derivative simplifies down to one term when evaluated at the critical point, specifically when $W_r=0$ is substituted in. Now it is easy to show that the last second order condition for a welfare maximum is met, $W_{rr}W_{qq} - W_{rq}^2 > 0$. This must be true if $r^* > 0$, $Q^* > 0$, $c'(r^*) > 0$, $D'(r^*Q^*) > 0$, and $P'(Q^*) < 0$.

$$\begin{aligned} W_{rr}W_{qq} - W_{rq}^2 &= Q^*[-c''(r^*) - Q^*D''(r^*Q^*)][P'(Q^*) - r^{*2}D''(r^*Q^*)] - [r^*Q^*D''(r^*Q^*)]^2 \\ &= -Q^*c''(r^*)P'(Q^*) - Q^{*2}D''(r^*Q^*)P'(Q^*) + r^{*2}Q^*c''(r^*)D''(r^*Q^*) \\ &\quad + r^{*2}Q^{*2}[D''(r^*Q^*)]^2 - r^{*2}Q^{*2}[D''(r^*Q^*)]^2 \\ &= -Q^*c''(r^*)P'(Q^*) - Q^{*2}D''(r^*Q^*)P'(Q^*) + r^{*2}Q^*c''(r^*)D''(r^*Q^*) \\ &> 0 \end{aligned}$$

The final second order condition must be met because every term in the final line above must be positive. Therefore the critical r^* and Q^* values defined by the first order conditions are indeed consistent with a welfare maximum.

Table of theoretical predictions

Variable	Predictions
Emission rate	$r^* = r^a = r^c < r^0$
Output quantity	$Q^* = Q^a < Q^c < Q^0$
Output price	$P(Q^0) < P(Q^c) < P(Q^a) = P(Q^*)$
Total emissions	$E^* = E^a < E^c < E^0$
Emission permit price	$P_c = P_a = D'(r^*Q^*)$

Appendix B: A Multiple-Agent Model

With the same functional assumptions as in the last section, this section uses a model with ‘n’ firms with possibly different emission rates, output, and variable and capacity cost functions in order to illustrate the impact that differential firms have on the theoretical predictions of the model. Again, we will commence with the optimal case. A social planner would want to choose emission rate and output for each firm in order to maximize total welfare, and so the social planner’s welfare maximization problem is

$$\text{Max}_{\{r_i, q_i\} \forall i} \text{Total Welfare} = \int_{z=0}^Q P(z) dz - \sum_{i=1}^n c_i(r_i) q_i - w_i q_i - D\left(\sum_{i=1}^n r_i q_i\right), \quad (\text{B1})$$

where q_i is the output of firm i and $Q = \sum_{i=1}^n q_i$. The first order conditions are

$$-c'_i(r_i^*) = D'\left(\sum_{i=1}^n r_i^* q_i^*\right) \text{ for } \forall i \quad (\text{B2})$$

when q_i is greater than zero and

$$P(Q^*) = c_i(r_i^*) + w_i + r_i^* D'\left(\sum_{i=1}^n r_i^* q_i^*\right) \text{ for } \forall i \quad (\text{B3})$$

The first set of conditions says that the optimal planner would set emission rates, the r_i^* s, such that each firm’s marginal abatement cost must equal the marginal damage of aggregate emissions. The latter set say that output levels must be set such that each firm’s marginal cost including the externality must be equal to the price of output at Q^* . Aggregate emissions will be

$E^* = \sum_{i=1}^n r_i^* q_i^*$. Since equation sets B2 and B3 do not contain q_i^* as an isolated term, only the

total Q^* and E^* is identified in the solution, then any combination of q_i^* s that sum to Q^* , and that sum to E^* when multiplied by the corresponding r_i^* , is a possible solution to the welfare maximization problem. This means that output quantity distribution is not only indeterminate between like firms, but is not even determined between firms with different cost structures.

For the remaining analysis we will assume that each firm’s cost function and variable cost is exactly the same as was implicitly assumed in the optimal planners problem, that way we can compare each firm’s behaviour under different scenarios. Each unregulated competitive firm’s profit maximizing problem and solution looks very similar to our earlier analysis and again provides a useful benchmark:

$$\text{Max}_{\{r_i, q_i\}} \pi_i^0 = P(Q)q_i - c_i(r_i)q_i - w_i q_i. \quad (\text{B4})$$

The two first order conditions are

$$-c_i'(r_i^0) = 0, \quad (\text{B5})$$

as long as output is greater than zero, and

$$P(Q^0) = c_i(r_i^0) + w_i. \quad (\text{B6})$$

Equation B5, which states that each firm sets its marginal abatement cost to zero, will ensure that each firm chooses an emission rates higher than it's optimal rate, $r_i^0 > r_i^*$, since $-c_i'(r_i^0) = 0 < -c_i'(r_i^*) = D'(E^*)$ and marginal abatement cost is monotonically decreasing in 'r'. Since $c_i(r_i^0) < c_i(r_i^*)$ and equation B6 is missing the positive marginal damage term that the social planner's first order condition contains in equation B3, we know that the right hand side of equation B6 must be smaller than that of equation B3 and so the uncontrolled output price must be smaller than the optimal output price, $P(Q^0) < P(Q^*)$. This directly implies that uncontrolled output is greater than optimal output, $Q^0 > Q^*$. Since only the total output is identified in the uncontrolled equilibrium, any set of q_i s that sum to Q^0 will be equilibrium quantities. Notice in this general case that there are no conditions on aggregate emissions, the sum of the individual ' $r_i^0 q_i^0$ ' firm emissions is unknown since the distribution of output between firms is unknown. Since every firm has a higher uncontrolled emission rate than their optimal rate, we do know that aggregate uncontrolled emissions must be higher than optimal emissions, no matter how this higher level of aggregate output is distributed. However if the firm's cost functions have the property that each firm's marginal abatement cost is equal to zero at the same emission rate, $r_{\max}^0 = c_i'^{-1}(0)$ for all 'i', then aggregate emissions must be $E^0 = r_{\max}^0 Q^0$.

Under a cap-and-trade scenario firm 'i's profit maximization problem is just

$$\text{Max}_{\{r_i, q_i\}} \pi_i^a = P(Q)q_i - c_i(r_i)q_i - w_i q_i - P_a(r_i q_i - A_i). \quad (\text{B7})$$

The two first order conditions are

$$-c_i'(r_i^a) = P_a \quad (\text{B8})$$

if q_i is greater than zero and

$$P(Q^a) = c(r_i^a) + w_i + r_i^a P_a. \quad (B9)$$

Equation B8 ensures that each firm's emission rate is set such that their marginal abatement cost is equal to the price of allowances. Equation B9 makes certain that each firm earns zero profit. As in the representative agent model, as long as the regulator allocates the optimal total number

of allowances, $\sum_{i=1}^n A_i = E^*$, then the cap-and-trade equilibrium looks just like the optimal

equilibrium above, where the price of allowances equal the optimal marginal damage (equations B2 and B3, and B8 and B9 are of the exact same form). This means that under a cap-and-trade system each firm will set their emission rate to the optimal rate and aggregate output, output price and aggregate emissions will also be at their optimal levels. Again, the equilibrium distribution of output quantity between firms is unidentified, but all possible equilibrium distributions are optimal.

Lastly, we focus again on a price-taking baseline-and-credit firm, only now with the possibility that different firms have different cost structures. Firm 'i's profit maximization problem is

$$\underset{\{r_i, q_i\}}{\text{Max}} \pi_i^c = P(Q)q_i - c_i(r_i)q_i - w_i q_i - P_c q_i (r_i - r^s), \quad (B10)$$

where $q_i(r_i - r^s)$ is firm 'i's net demand for credits. Assuming that q_i is greater than zero, the two first order conditions are

$$-c_i'(r_i^c) = P_c \quad (B11)$$

and

$$P(Q^c) = c_i(r_i^c) + w_i + r_i^c P_c - r^s P_c. \quad (B12)$$

Equation B11 is the usual marginal cost of abatement equals to the price of credits condition and equation B12 is the usual zero-profit condition. Let us assume that the regulator sets the emission rate standard equal to the average emission rate under the social planner scenario, $r^s = E^*/Q^*$. If the emission standard is binding and gross demand for credits is equal to gross supply then

$\sum_{i=1}^n r_i^c q_i^c = \sum_{i=1}^n r^s q_i^c$. Substituting in for the emission standard we can calculate that

$$\sum_{i=1}^N r_i^c \frac{q_i^c}{Q^c} = \sum_{i=1}^N r_i^* \frac{q_i^*}{Q^*}. \text{ This last equation implies that if we assume that each firm's output stays}$$

in the same in proportion to aggregate output in the baseline-and-credit case as it was in the optimal case, then each firm choosing its optimal emission rate will satisfy our baseline-and-credit equilibrium condition. Focusing on the baseline-and-credit equilibrium in which each firm chooses their optimal emission rate, one notices that equation B12 is similar to the optimal equation B3 (since equation B11 together with the assumption that $r_i^c=r_i^*$ implies that $P_c=D'(E^*)$) except for an extra negative term in equation B12 that does not exist in equation B3. This will cause baseline-and-credit output price to be less than the optimal output price, implying that the aggregate output in this case will be larger than aggregate output chosen by the social planner. Each firm choosing its optimal emission rate combined with a proportionately larger than optimal aggregate output will result in aggregate emissions exceeding the optimal amount under this baseline-and-credit trading scheme.

Appendix C: Screen Shots

Allowance Order Form

Please enter your bid/ask schedule below:

You have **5909.26** dollars cash and **22** allowances in inventory.

Bids:

At \$ **12** I will buy **10** units,

at \$ **5** I will buy **10** units more,

at \$ **1** I will buy **0** units more.

I will buy a maximum of **20** units.

Costing a MAXIMUM of **120** dollars.

Asks:

At \$ **48** I will sell **12** units,

at \$ **72** I will sell **10** units more,

at \$ **73** I will sell **0** units more.

I will sell a maximum of **22** units.

Submit

Reset

DataView

Planner | CapacityRegistry | PermitRegistry | Market | Order | Ledger | BalanceSheet | IncomeStatement

Wizard

If I have a capacity of **10** units with an emission rate of **2** then total plant costs will be **740.74** dollars that will last for **1** period(s).

If I produce and sell **10** units of output per period, I will be operating at **100** % of capacity, and if allowances are worth **10** dollars/allowance,

then EACH unit of output will cost **20** dollars for required allowances, **35.00** dollars in materials, and **74.07** dollars in average plant costs, leaving my average total cost per unit, per period, being **129.07** dollars.

Current Period's Capacity and ER

Last Period's Capacity: **10**

Last Period's Emis. Rate: **2**

Capacity: **10**

Emission rate: **-**

Life span: **1**

Remaining life: **-**

Total plant cost: **-**

Output

Demand intercept: **200**

Demand slope: **0**

Market price last period: **200**

Market volume last period: **10**

Output you sold last period: **10**

Output materials cost: **35**

Cash

Current Cash: **5909.26**

Allowances

Market price last period: **10**

Market volume last period: **0**

Allowances bought last period: **0**

Current allowances inventory: **22**

Subject 1 Period 2

Please make your allowance market order

Subject: 1 | Period: 2 | Cumulative Payoff: \$954.26 lab dollars

OutputOrder

Please fill out the output order form below:

At a price of:

I will sell:

\$ 46 10 units

\$ 47 0 units more

\$ 48 0 units more

I will sell a maximum of 10 units.

I have a maximum capacity of 10 units of output.

If I sell this many units of output it will require 10

allowances and I have 33 right now.

DataView

Planner | CapacityRegistry | PermitRegistry | Market | Order | Ledger | BalanceSheet | IncomeStatement

Filter: Subject: 1 End of Period: 2

Income Statement for the end of period 2

Revenue:

Output Sales	10 units @	200.00 each	=	2000.00
Allowance Sales	0 units @	12.00 each	=	0.00
Allowances Endowed	21 units @	10.00 each	=	210.00

Total				= 2210.00

Cost of Goods Sold:

Materials Used	10 units @	35.00 each	=	350.00
Allowances Used	10 units @	11.59 each	=	115.91
Cost of Allowance Sales	0 units @	0.00 each	=	0.00

Total				= -465.91

Operating Expenses:

Depreciation on Plant	=	-1025.93
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Net Income for the Period = 718.16
=====

Basic Data

Capacity = 10 units Emission Rate = 1 per unit of output
Output = 10 units Emissions = 10 units

Allowance Inventory

Opening Balance this period = 1

Subject 1 Period 3

Please make your output market order

Subject: 1 Period: 3 Cumulative Payoff: \$1,672.42 lab dollars

CapacityOrderForm

CHOOSE CAPACITY:

You now have to choose a capacity that will last for the next period(s).

Last period you had a capacity of units.

Last period you sold units of output.

I choose to build a capacity of

units for the next period(s).

Submit

DataView

Planner | CapacityRegistry | PermitRegistry | Market | Order | Ledger | BalanceSheet | IncomeStatement

Filter: Subject: Period: Account:

PERIOD	DESCRIPTION	ACCTNAME	DEBITAMT	CREDITAMT	BALANCE
3	Endowment for period	Allowance Inventory	252	0	391.09
3	Endowment for period	Allowances Received	0	252	252
3	Build Capacity	Fixed Plant	1025.93	0	1025.93
3	Build Capacity	Cash	0	1025.93	5507.41
3	Sold Output	Cash	2000	0	7507.41
3	Sold Output	Output Sales	0	2000	2000
3	Sold Output	Materials Expenses	350	0	350
3	Sold Output	Cash	0	350	7157.41
3	Sold Output	Allowance Expenses	118.51	0	118.51
3	Sold Output	Allowance Inventory	0	118.51	272.58
3	Depreciate Capacity	Depreciation Expenses	1025.93	0	1025.93
3	Depreciate Capacity	Accumulated Depreciation	0	1025.93	-1025.93
3	Plant fully depreciated	Accumulated Depreciation	1025.93	0	0
3	Plant fully depreciated	Fixed Plant	0	1025.93	0
3	Close Output Sales	Output Sales	2000	0	0
3	Close Output Sales	Net Income	0	2000	2000
3	Close Allowance Sales	Allowance Sales	0	0	0
3	Close Allowance Sales	Net Income	0	0	2000
3	Close Allowances Receive	Allowances Received	252	0	0
3	Close Allowances Receive	Net Income	0	252	2252
3	Close Materials Expenses	Net Income	350	0	1902
3	Close Materials Expenses	Materials Expenses	0	350	0
3	Close Allowance Expenses	Net Income	118.51	0	1783.49
3	Close Allowance Expenses	Allowance Expenses	0	118.51	0
3	Close Depreciation Expen	Net Income	1025.93	0	757.56
3	Close Depreciation Expen	Depreciation Expenses	0	1025.93	0
3	Close Cost of Allowance	Net Income	0	0	757.56

Subject 1 Period 3

Please make your capacity choice

Subject: 1 | Period: 3 | Cumulative Payoff: \$2,429.99 lab dollars