- My discussion assumes that you have studied the material in King-Rebelo previously and are quite familiar with it. Please consult the paper as well as your 723 notes on the business cycle and growth models to make sure you understand everything. This introductory material is crucial to following the rest of the material covered in the course.

- Large number infinitely lived agents who have identical expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, L_t) \;\; \beta > 0$$

- β is discount factor, c_t is consumption in period t, and L_t is leisure in period t.

- All of this is based on information in time zero.

- Assume a constant population.

- U(.) is concave and increasing in c and L

  Two commonly used specifications:

$$U(c, L) = log c_t + x log L_t$$

  and

$$\frac{1}{1-\sigma}\left[c_t v(L_t)\right]^{1-\sigma} - \frac{1}{1-\sigma}, \sigma > 0, \sigma \neq 1$$

- See appendix in King and Rebelo 2000 for a discussion or KPR 1988.

  These preferences imply agents will want to "smooth consumption and leisure"

  Also these imply "intertemporal substitution of c and L" as the costs of c and L vary over time.

- **Endowments**:

  One unit of time spent on working or on Leisure

  ie.

$$N_t + L_t = 1 \qquad (2)$$

  Technology:

- Two inputs: Capital and Labour.

$$Y_t = A_t F(K_t, N_t) \quad (3)$$

  Ignoring labour augmenting technical progress to focus on business cycle issues.

- A_t is a random shock to productivity
- $F(.)$ is twice continously differentiable concave and homogenous of degree one.

  F satisfies the Inada Condition:
- $lim_{K\to\infty} F_K = 0$ and
- $lim_{K\to 0} F_K = \infty$

  Single good economy, which can be used for consumption or investment.
- Resource constraint:

$$Y_t = c_t + I_t \qquad (5)$$

  No government!
- Stock of capital evolves according to

$$K_{t+1} = I_t + (1-\delta)K_t \qquad (6)$$

  δ is the rate of depreciation.
- Initial conditions: $K_0 > 0, A_0 > 0$.
- The optimal control problem:

$$maxE \sum_{t=0}^{\infty} \beta^t u(c_t, L_t) \qquad (7)$$

  subject to (2) to (6)
- (3), (5) and (6) can be combined to get:

$$c_t + K_{t+1} = A_t F(K_t, N_{t)} + (1-\delta)K_t \ (8)$$

- The optimal path of capital accumulation can be got by choosing sequences for

  $\{C_t\}_0^{\infty}, \{L_t\}_{t=0}^{\infty}, \{N_t\}_{t=0}^{\infty}$ and$\{K_{t+1}\}_{t=0}^{\infty}$

  to max (7) s.t. (8) and (2)
- Write the Langrangean as:

$$\begin{aligned} L &= E[\sum_{t=0}^{\infty} \beta^t u(c_t, Lt) + \sum_{t=0}^{\infty} \beta^t \lambda_t \\ &\quad [AF(K_t, N_t) + (1-\delta)K_t - c_t - K_{t+1}] \\ &\quad + \sum_{t=0}^{\infty} \beta^t \omega_t [1 - L_{t-} N_t]] \end{aligned}$$

- **F.O.C.**

$$c_t : U_{c_t} = \lambda_t \qquad (9)$$

$$L_t : U_{L_t} \;= \omega_t \qquad (10)$$

$$N_t : \lambda_t A_t F_{N_t} = \omega_t \; (11)$$

$$K_{t+1} : \beta E \lambda_{t+1} \left[A_{t+1} F_{K_{t+1}} + 1 - \delta \right] = \lambda_t \; (12)$$

- **Interpretation:**
- Set Marginal Utility of consumption equal to its shadow price and MU of leisure equal to its shadow price.

  (11) states that you should allocate your time to work to ensure that the marginal benefit equals marginal cost expressed in utility terms.

  (12) states that the marginal value of a unit of consumption foregone should equal the PDV of the extra output made available for consumption (potentially next period).
- Remember these conditions hold for all time periods 't'.
- An optimal program will satisfy these FOC, the intial conditions and the oringial constraints as well as the transversality condition:

$$lim_{t \to \infty} \beta^t \lambda_t K_t = 0.$$

- Note that the choices of c_t, N_t and K_t are jointly made for all t.
- The optimal decisions respect the resource constraints of the economy. Why? Because take into account the shadow prices w_t and λ_t associated with our two constraints.
- So also solve for w_t and λ_t.
- Basic equivalence of this problem with the competitive market outcome.
- H.W. : Check that you get the same result for the market outcome by setting up the decentralized version of the model and deriving the FOC's. In other words: consider a prototypical decentralized real business cycle model with a large number of infinitely lived consumers who have preferences over consumption and leisure and have an endowment of one unit of time each period which they can devote to leisure or to work. The economy also has a large number of competitive firms who hire labour and physical capital inputs from consumers in return for wages and interest payments.These firms have access to a technology that uses these inputs to produce a good that

can be used for consumption or investment. Consumers buy this good in a competitive market for consumption and to invest in their stock of physical capital which depreciates at a constant rate. The number of consumers in the economy is growing at a constant rate.

- There exists a unique steady state in which the value of all variables can be uniquely determined.

- HW2: Characterize the steady state associated with the above optimal control problem for both set of preferences given above. Distinguish between the concepts of equilibrium, steady state and balanced growth path.

  TRANSITIONAL DYNAMICS

  Look at figure 6 in King-Rebelo (2000). All variables are in % deviation from their corresponding stationary values.

- The capital stock is 1% below SS level.

  Recall capital is the state variable in the system.

  Since K is low, individuals work harder and produce more output.

  This output is put towards capital accumulation

  ⇒ C is below ss

  ⇒ I is above ss

  Note that below ss K ⇒ high MP_K and high real rate of return on investing.

  ⇒ High real interest rate in market

  ⇒ High return acts as an "allocative signal" to postpone consumption.

  Why? Let

$$U(c, L) = log c_t + \chi \log L_t$$

⇒

$$U_c = \frac{1}{c_t}, U_L = \frac{\chi}{L_t}$$

So (9) ⇒

$$\lambda_t = \frac{1}{c_t}$$

and (12) ⇒

$$\frac{c_{t+1}}{\beta c_t} = \left[A_{t+1} F_{K_{t+1}+} 1 - \delta\right]$$

- when K_{t+1} is low relative to SS.⇒ RHS is high so that c_t is low relative to c_{t+1}

  Since K_t monotonically rises to reach its SS level, c will also monotonically rise to reach SS level.

  ⇒ c is initially below SS.

- Leisure behaves like consumption because it too is driven by the high return on capital.
- Transitional dynamics helpful in understanding the model but not the full story. Also helpful for analyzing impulse responses later.
- **Introducing Shocks:**
- Use the solow residual to figure out the stochastic compnent of TFP.
- show that "technology shocks" can account for a lot of the fluctuation observed in the data.

$$\log A_t = \rho \log A_{t-1} + \varepsilon_t$$

$0 < \rho < 1$ $\quad$ ρ is AR1 parameter, ε_t is $iid(0, \sigma_\varepsilon^2)$

(see 723 notes and K-R) on solow residual, calibration, Quickly review results of this model.