Asset Accumulation and Short Term Employment

Martin Browning
University of Copenhagen
Denmark

Thomas Crossley
University of York
Canada

Eric Smith
University of Essex
England

Address for correspondence:
Department of Economics
University of Essex
Wivenhoe Park
Colchester, Essex CO4 3SQ
ENGLAND
esmith@essex.ac.uk

July 2000
Introduction

This paper characterizes the search and consumption behavior of workers as they move between readily available low wage employment and uncertain high wage job search. In most studies of search behavior, job seekers are assumed to have risk neutral preferences and hence maximize the expected utility of the present discounted value of income less costs. This specification implies that the pattern of consumption is either indeterminate or completely insured. It also generates a stationary reservation wage strategy. A wage offer rejected today will not be acceptable in the future and any accepted job opportunity will always be preferred to further search. (See Mortensen, 1986). As a result, the transition rate from unemployment into employment does not vary over time or with differences in wealth. In addition, workers only leave a job if they find a better opportunity (through on the job search) or if the employment relationship changes in some way so that the payoff to continued work becomes less productive than the return to search.

For risk averse job seekers without insurance, this characterization does not hold. In this case, the pattern of consumption becomes well defined. Individuals consume more as their assets increase. In addition, the job acceptance decision also depends on the individual’s asset levels although further restrictions on the degree of risk aversion are needed to characterize this relationship.\footnote{Job seekers with decreasing absolute risk aversion become pickier as assets increase (the reservation wage declines with asset holdings). As a result, rich individuals search longer and hence tend to match with higher paying jobs (Danforth, 1979). In addition, as risk averse job seekers consume from assets and become less picky, they may on occasion want to recall previously rejected offers (Hall, Lippman and McCall 1979).} Employment, however, is again permanent - as with risk neutral job search, unless something changes for the worker, there are no job to unemployment transitions.

Credit constraints, as this paper demonstrates, can give rise to employment-to-unemployment turnover. Without the ability to borrow or insure, risk averse individuals become willing to accept readily available, low wage “bad” jobs in order to accumulate assets which subsequently fund search for high wage “good” jobs. If the ensuing search for high wage employment is unsuccessful and assets become depleted, these individuals then take up bad jobs again and repeat the cycle. Bad jobs act as a floor or safety net for those who are unsuccessful at job search and do not have the ability to borrow.

As a result, temporary employment in bad jobs generates quits without shocks to productivity or the job search process. The pattern of asset accumulation and consumption follows accordingly. While working for low pay,
workers save a constant fraction of their wage. At the time of a quit, consumption starts to decline until either a job is found or the worker returns to another low paying job.

Job seekers would in general prefer borrowing against future income to fund consumption during search rather than using low wage work to build assets. Credit constraints prevent such behavior and hence take on an central role in the determination of income distribution. Rather than limit human capital investment as often discussed in the literature, here they restrict the extent of search and thereby the flow of workers into desirable high wage jobs. Consumption behavior reflects the extent of this effect. Although consumption differs for searchers and low paid employees, these two types appear relatively similar when compare with workers who have acquired high paying jobs.\textsuperscript{2}

The analysis further reveals that turnover costs play a key role in determining asset accumulation as well as the durations of employment and job search. In particular, rising turnover costs lead to a longer employment spells in low wage jobs. While in these bad jobs, individuals save a higher level assets but at a slower rate. When they do search, workers are more cautious. They run down assets at a slower rate by consuming less and thereby searching longer.

A framework for endogenous job separation

Consider an economy in which workers can at any time accept a low paying job at a wage $w_L > 0$. Such jobs are always available. Individuals who forgo the low wage sector (and only these workers) can search for higher paying employment with the associated wage $w_H > w_L$. When looking for a job, a job seeker receives a high wage offer with probability $\alpha$.

Income comes from two other sources as well. In all periods the worker receives a fixed flow of income $w_F \geq 0$ which can be thought of as (the individual’s allowance of) family income. During periods of unemployed job search, the worker also receives a payment of $0 < b < w_L$. This income has a number of interpretations. It can be thought of as the return from leisure, home production, and/or unemployment benefits of infinite duration. (We will later consider payments of $b$ for a finite duration.)

While employed and during the job search process, a worker chooses consumption to maximize expected lifetime utility. Let $u(\cdot)$ represent the

\textsuperscript{2}Preliminary evidence from Canada on separated workers supports such behavior.
worker’s risk averse preferences in each period. $u(\cdot)$ is continuous, differentiable and bounded above with the standard Inada properties so that $u'(\cdot) > 0$, $u''(\cdot) < 0$, $u(0) = 0$, $u'(0) = \infty$, and $u'(\infty) = 0$.

Time is continuous and infinite. Suppose that at date $t$ a worker has assets $A_t \geq -F$ where $F \geq 0$. A worker entering the market for the first time does so with assets $A_0 \geq 0$. Job seekers earn (or pay) interest on these assets at rate $r$ which also equals the individual’s rate of time preference.

Although it is costless to accept a low wage job, it is not costless to embark on search. Entrants into the market who decide to search immediately as well as low wage workers who quit employment in order to search pay a turnover cost $K > 0$. Turnover costs have a number of interpretations. For transitions from low wage employment, the most immediate corresponds to an exit cost borne by the worker. In the context presented here, an exit cost of this sort is equivalent to an entry fee in the low wage sector properly adjusted in present value terms. An alternative view is that these costs represent (round trip) transportation costs between spatially distinct sectors, the low wage - full employment and the high wage with search unemployment sectors. This perspective highlights the similarities of this model with that of Harris and Todaro (1970). Here, however, the economy explicitly accounts for the dynamic flows between sectors. Of course, the act of changing sectors need not be explicitly spatial but nonetheless involve a transportation cost.

### Job search

Consider first the “partial” problem of a job seeker (an individual in the job search process) with assets $A_D$ which are for now taken as given. $A_D$ may differ from initial assets $A_0$ if the worker initially participates in the low wage sector. While looking for high wage employment, the worker’s problem is to choose the maximum duration of search $T$ (given a job is not found), a consumption path, $c_t$, for $t \in [0, T]$, and the asset level desired at the end of the search period, $A_T$. These decisions are made bearing in mind the

---

1. The familiar no-debt convention in which the credit constraint parameter equals zero, $F = 0$, will be used throughout. At this point, however, the constraint is parameterized in order to qualitatively assess the effects of credit rationing.

2. and hence an entry fee is not equivalent to an exit cost.

The transportation cost interpretation can differ somewhat from the entry and exit fee perspective. As discussed below, a worker with sufficiently high assets will initially search. In this case, there is no exit (or entry) cost from the low wage job to correspond to the transportation fee. However, as the focus here is on flows between job search and low wage employment, this case is of minor concern.
opportunities of low wage employment, represented here by the value of low wage employment, $V(A)$.

As high paid jobs last forever, a worker with assets $A_t$ in a high wage job will optimally consume $rA_t + w_H + w_F$ indefinitely. The corresponding value of high wage employment is therefore $u(rA_t + w_H + w_F)/r$. As a result, the searching worker’s problem can be written as\(^5\)

\[
W(A_D) = \max_{c_t, T, A_T} \int_0^T u(c_t)e^{-(r+\alpha)t}dt + \frac{\alpha}{r} \int_0^T u(rA_s + w_H + w_F)e^{-(r+\alpha)s}ds
\]

\[+ e^{-(r+\alpha)T}V(A_T)\]

subject to

(i) \[A_D + \int_0^T (w_F + b - c_t)e^{-rt}dt - e^{-rT}A_T \geq 0\] (Budget Constraint)

(ii) \[e^{-rT}[A_T + F] \geq 0\] (Credit Constraint)

where

\[A_s = e^{rs} \left[ A_D + \int_0^s (w_F + b - c_v)e^{-rv}dv \right] \geq 0\]

\[0 \leq s \leq T\]

\(^5\)The objective function can be viewed as the limit of the discrete time problem. Given a time period of length $\Delta$, the worker receives the expected payoff

\[
W(A) = u(c_0)\Delta + \sum_{t=1}^{T} \left( 1 - \frac{\alpha\Delta}{1 + r\Delta} \right)^t u(c_t)\Delta + \frac{\alpha\Delta}{1 - \alpha\Delta} \sum_{t=1}^{T} \left( 1 - \frac{\alpha\Delta}{1 + r\Delta} \right)^t u(rA_t + w_H)/r
\]

\[+ \frac{1}{1 - \alpha\Delta} \left( \frac{1 - \alpha\Delta}{1 + r\Delta} \right)^T V(A_T)\]

Letting $\Delta \to 0$ yields the continuous time objective function.
To complete the specification of the worker’s decision problem, we next characterize the value of working in low wage jobs, $V(A)$. Workers in low wage employment have the option of turning at some time to high wage job search at a cost $K$ or remaining indefinitely in low wage employment with no intention of further search. A worker in low wage employment with initial assets $A_0$ chooses a duration of employment $D$, a consumption pattern $\zeta_t$, $t \in [0, D]$, and a terminal level of assets $A_D$. If the worker turns to high wage job search ($D < \infty$), terminal assets (along with $w_F$ and $b$) fund the turnover cost $K$ and the subsequent consumption while searching.

The worker’s problem is thus expressed by\(^6\):

\[
V(A_0) = \max \int_0^D u(\zeta_t)e^{-rt}dt + e^{-rD}W(A_D)
\]

subject to

\[(i) \quad A_0 + \int_0^D (w_F + w_L - \zeta_t)e^{-rt}dt - e^{-rD}A_D - e^{-rD}K = 0 \quad (\text{Budget Constraint})\]

\[(ii) \quad e^{-rD}(A_D - K + F) \geq 0 \quad (\text{Credit Constraint})\]

Substituting in for $W(A_D)$ as well as for $A_D$ in the budget constraint yields the following problem

\[
V(A_0) = \max \int_0^D u(\zeta_t)e^{-rt}dt + e^{-rD} \left\{ \int_0^T u(c_t)e^{-(r+\delta)t}dt \right. \\
+ \frac{\alpha}{r} \int_0^T u(rA_s + w_H + w_F)e^{-(r+\delta)s}ds + e^{-(r+\delta)T}V(A_T) \right\}
\]

\(^6\)Note as well that there are nonnegativity constraints on the choice variables although this is a potentially binding concern only for the duration of low wage employment ($D \geq 0$) as discussed below.
subject to

\[ (i) \quad A_0 + \int_0^D (w_F + w_L - \zeta_t)e^{-rt}dt + e^{-rD} \int_0^T (w_F + b - c_t)e^{-rt}dt \]
\[ -e^{-rD}K - e^{-r(D+T)}A_T \geq 0 \]

and

\[ (ii) \quad e^{-r(D+T)}(A_T + F) \geq 0 \]

where

\[ A_s = e^{rs} \left\{ e^{rD}A_0 + e^{rD} \int_0^D (w_F + w_L - \zeta_v)e^{-rv}dv \right. \]
\[ + \left. \int_0^s (w_F + b - c_v)e^{-rv}dv - K \right\} \geq 0 \quad 0 \quad 0 \quad s \quad T \]

For multipliers \( \mu_1 \) and \( \mu_2 \) associated with constraints (i) and (ii) respectively, the first order conditions with respect to \( \zeta_t, c_t, D, T, A_T \) are\(^7\)

\[ e^{-rt} \left\{ u'(\zeta_t) - \alpha \int_0^T u'(rA_S + w_H + w_F)e^{-\alpha s}ds - \mu_1 \right\} = 0 \quad t \in [0, D] \quad (1) \]

\(^7\)The asset equation for \( A_s \) implies that

\[ \frac{\partial A_s}{\partial \zeta_t} = -e^{r(D+s-t)} \quad 0 \quad s \quad T, \quad 0 \quad t \quad D \]

\[ \frac{\partial A_s}{\partial c_t} = \begin{cases} 
- e^{r(s-t)} & s \geq t \\
0 & s < t \end{cases} \quad 0 \quad s, t \quad T \]

\[ \frac{\partial A_s}{\partial D} = e^{rs} \left[ re^{rD}A_0 + re^{rD} \int_0^D (w_F + w_L - \zeta_v)e^{-rv}dv + w_F + w_L - \zeta_D \right] \]

\[ \frac{\partial A_s}{\partial T} = \frac{\partial A_s}{\partial A_T} = 0 \]
\[ e^{-r(D+t)} \left\{ u'(c_t)e^{-\alpha t} - \alpha \int_0^T u'(rA_s + w_H + w_F)e^{-\alpha s} ds - \mu_1 \right\} = 0 \]  
\quad t \in [0,T] \tag{2}

\[ e^{-rD} \left\{ u(\varsigma_D) - r \int_0^T u(c_t)e^{-(r+\alpha)t} dt - \alpha \int_0^T u(rA_s + w_H + w_F)e^{-(r+\alpha)s} ds \right\} - re^{-(r+\alpha)T}V(A_T) + \alpha \int_0^T u'(rA_s + w_H + w_F)(\partial A_s / \partial D)e^{-(r+\alpha)s} ds 

+ \mu_1 \left[ e^{-rT} w_F - (1 - e^{-rT})b + w_L - \varsigma_D + r \int_0^T c_t e^{-rt} dt + rK + re^{-rT}A_T \right] - \mu_2 r e^{-rT}(A_T + F) = 0 \tag{3}

\[ e^{-r(D+T)} \left\{ u(c_T)e^{-\alpha T} + \frac{\alpha}{r} u(rA_T + w_H + w_F)e^{-\alpha T} - (r + \alpha)e^{-\alpha T}V(A_T) \right\} + \mu_1 (w_F + b - c_T + rA_T) - \mu_2 r(A_T + F) = 0 \tag{4}

\[ e^{-r(D+T)} \{ e^{-\alpha T} V'(A_T) - \mu_1 + \mu_2 \} = 0 \tag{5} \]

while the Kuhn Tucker conditions for the constraints (i) and (ii) are
\[ \mu_1 \left\{ A_0 + \int_0^D (w_F + w_L - \zeta_t)e^{-rt}dt + e^{-rD} \int_0^T (w_F + b - c_t)e^{-rt}dt ight\} = 0 \]  

(6)

\[ -e^{-rD}K - e^{-r(D+T)}A \]  

(7)

\[ \mu_2 e^{-r(D+T)}(A_T + F) = 0 \]  

(8)

\[ e^{-rD} \left\{ u'(c_t)e^{at} - \alpha \int_t^T u'(rA_s + w_H + w_F)e^{-as}ds - \mu_1 \right\} = 0 \]  

(9)

Consumption Behavior

From equation (1), consumption while employed (that is over the period \(0 \leq t \leq D\)) is constant.\(^8\) Substituting in \(\bar{c} = \zeta_t\) for all \(t \in [0, D]\), adding in \(\partial A_s/\partial D\) and rearranging terms, equations (1) and (2) simplify to\(^9\)

\[ u'(\bar{c}) - \alpha \int_0^T u'(rA_s + w_H + w_F)e^{-as}ds - \mu_1 = 0 \]  

(8)

\[ e^{-rD} \left\{ u'(c_t)e^{-at} - \alpha \int_t^T u'(rA_s + w_H + w_F)e^{-as}ds - \mu_1 \right\} = 0 \]  

\(t \in [0, T]\)

---

\(^8\)In the adopted notation here, the index \(t\) (or for that matter \(s\)) does not necessarily correspond to chronological time. For consumption during low wage employment, \(t\) does at first match real time but for consumption while searching, \(c_t\), this index differs from the date by \(D\). Of course, if cycles of work and employment occur, the index further differs by a factor of \(T + D\).

\(^9\)The budget constraint holds \((\mu_1 > 0)\) and by assumption A2 below, search duration, if any, is finite \((T < \infty)\). Both outcomes allow for the elimination of common factors involving these terms. On the other hand, we maintain the possibility that \(D = \infty\) or \(D = 0\) and avoid eliminating factors involving these terms whey the equal zero.
From (9), $c_t$ is strictly positive for all $t$ and decreasing over time. As search proceeds, consumption falls until either a high wage job is found or search terminates with corresponding consumption $c_T$. From equation (8) and equation (9) for $t = 0$, consumption at the beginning of search equals consumption during low wage employment (if it is taken on so that $D > 0$) which in turn is less than or equal to the low wage plus family income:

$$c_0 = \bar{c} - rA_0 + w_L + w_F.$$

Consumption during search can be characterized further. Provided that search occurs, that is $T > 0$, equation (9) generates the differential equation

$$\frac{dc_t}{dt} = r A_t + w_H + w_F - c_t$$

while the asset equation gives a second differential equation in assets and consumption

$$\frac{dA_t}{dt} = rA_t + w_F + b - c_t$$

The associated phase diagram along with the equations for $\frac{dA_t}{dt} = 0$ and $\frac{dc_t}{dt} = 0$ is illustrated in Figure 1. Since the stationary lines for $\frac{dA_t}{dt} = 0$ and $\frac{dc_t}{dt} = 0$ are parallel at a distance of $w_H - b$ from each other, there are no stationary points in this system. Moreover, it is straightforward to establish that the optimal solution lies between these two lines:

$$rA_t + w_F + b < c_t < rA_t + w_H + w_F.$$

As a result, for any terminal point $(c_T, A_T)$, there is a unique path and any stable path has decreasing assets and consumption over time.
Search and Employment Duration

The first order equations (3)-(7) can be simplified in a similar fashion giving:

\[ u(\zeta) - rV(A_0) + (rA_0 + w_F + w_L - \zeta) \alpha \int_0^T u'(rA_s + w_H + w_F)e^{-\alpha s} ds \quad (12) \]

\[ + \mu_1 (rA_0 + w_F + w_L - \zeta) = 0 \]

\[ e^{-rD} \left\{ u(c_T) + \frac{\alpha}{r} u(rA_T + w_H + w_F) - (r + \alpha)V(A_T) \right. \]

\[ + \left. \mu_1 e^{\alpha T}(w_F + b + rA_T - c_T) \right\} = 0 \quad (13) \]
The solutions of primary interest are those in which workers move back and forth between low wage employment and job search until finding a permanent high wage job. To establish that such solutions occur, we rule out alternatives that do not fit this pattern. To ease notation, restrict attention to a baseline case in which the family income and unemployment benefit parameters both equal zero and the credit constraint does not allow any borrowing: \( w_F = b = F = 0 \).

For an arbitrary level of initial assets \( A_0 \geq 0 \), note that \( D = \infty \) along with \( \zeta_t = \bar{\zeta} = rA_0 + w_L \), is a solution to equations (8)-(16). In this solution, \( c_t, T, A_T \) are undetermined as search does not occur. The following claim establishes conditions on wages and turnover costs under which this solution to the first order conditions is not optimal. Wages must be sufficiently high so that search is attractive while the turnover costs must be sufficiently small so that the worker is willing to participate in high wage job search.

**Claim 1** If

\[
\frac{\alpha}{r} (u(rA_0 + w_H) - u(rA_0 + w_L)) - u'(rA_0 + w_L)w_L > 0,
\]

then a worker will at some point switch to high wage job search \( (D < \infty) \) given sufficiently small \( K \).

**Proof:** See Appendix

On the other hand, there may be no transition from search into low wage employment. If search is sufficiently attractive, it may be optimal to search indefinitely, \( T = \infty \). In this case, the job seeker runs down assets (recall that \( c_t > rA_t \)) so that consumption becomes arbitrarily small as time proceeds. This action is ruled out when wages in good are not “too attractive.”
Claim 2 If \( \frac{\alpha}{r+\alpha} u(w_H) < u(w_L) \), then \( T < \infty \).

Proof: See Appendix

Given these two results, we impose the following assumptions on wages so that cycles can emerge:

**A1:** \( \frac{\alpha}{r} \left( u(rA_0 + w_H) - u(rA_0 + w_L) \right) - u'(rA_0 + w_L)w_L > 0 \)

**A2:** \( \alpha u(w_H) < (r + \alpha)u(w_L) \)

Notice as well that \( T = 0 \) is not part of an optimal plan as this strategy involves paying a transition fee without any possible payoff from search. On the other hand for some initial asset levels, low wage employment (accompanied by asset accumulation) may not be desirable. In other words, Claim 1 does not demonstrate that at some point a worker will necessarily take on (and then later quit) a low wage job so that a worker may forgo low wage employment and immediately search. Given \( A_0 \), it may be optimal to set \( D = 0 \).

Regardless of the initial choice of \( D \) and \( T \) (given \( A_0 \)), we now establish conditions such that at the end of unsuccessful search (which is non-trivial) an individual will have exhausted the assets in which case the credit constraint binds.

Claim 3 If

\[
\frac{\alpha}{r} \left( u(rA + w_H) - u(rA + w_L) \right) - u'(rA + w_L)w_L > 0 \quad \forall A \in [0, A_0]
\]

then \( A_T = 0 \)

Proof: See Appendix

Given the generalization of assumption **A2** in Claim 3, a worker who has just left job search will not immediately return to search (by setting \( D = 0 \)) when solving \( V(0) \): a worker will not exit search, take on a low wage job for

---

\[^{10}\text{While for assets less than } K \text{ this is not feasible since the subsequent search does not occur: } T = 0, \text{ for } A_0 \text{ greater than some critical value this will indeed be a solution. More specifically, if workers begin with different endowments, } A_0, \text{ those with high levels of initial assets will immediately search for high wages and only take up low wage jobs when high wage search is unsuccessful. On the other hand, workers with low endowments will accumulate assets before search (} D > 0 \text{).} \]
zero duration, and then search again after paying $K$. This is impractical and not feasible for $K > 0$.

As a result, the (repeated) pattern of low wage employment followed by high wage job search emerges. Cycles exist. Regardless of initial assets, workers will (with some probability) search until assets are used up. At this point, Claim 1 establishes that they will not take a low wage job permanently. Claim 2 establishes they will not search with zero assets. Instead they will take on low wage employment for a finite period after which they search. If unsuccessful, this search will terminate with zero assets at which point the process begins anew. In other words, the solution to $V(0)$, is such that workers cycle indefinitely between low wage employment and high wage job search until they ultimately find a high wage job.

**Turnover Costs**

How does consumption and the duration of job search respond to a change in turnover costs? Do workers search longer? Do they begin search with higher assets? What are the consequences for unemployment? Since asset accumulation and job quits occur only in the case where workers who terminate job search do so when the credit constraint binds ($A_T = -F = 0$), when considering these effects it is sufficient to concentrate on the case in which initial and terminal assets are zero.

As shown in the appendix, in this cycle consumption during low wage employment increases with turnover costs. This further implies that the initial level of consumption during search also rises:

$$\partial c_0 / \partial K = \partial \xi / \partial K > 0.$$ 

On the other hand, consumption at the end of high wage job search declines with turnover costs:

$$\partial c_T / \partial K < 0$$

These results, in turn, determine the changes in employment and search durations. Since the job search phase diagram for $c_t$ and $A_t$ is independent of turnover costs, an increased initial consumption $c_0$ along with decreased terminal consumption $c_T$ implies that the new solution is on lower path but with larger first period consumption. Given higher initial consumption, it follows that assets rise at the outset of high wage search.

On the lower trajectory, consumption for a given asset level falls:

$$\partial c(A_t) / \partial K < 0.$$
With more assets being consumed at a slower rate, the duration of search necessarily rises\textsuperscript{11}:

\[
\frac{\partial T}{\partial K} > 0.
\]

As consumption during low wage employment rises, the rate of asset accumulation declines. However, at the termination of the low wage job at time $D$, the worker starts high wage job search with higher consumption - recall that $c_0$ has increased. From the phase diagram, it follows that the worker must arrive with higher assets, $A_D$. To accumulate a larger asset level with a slower accumulation rate requires that the duration of low wage employment increases:

\[
\frac{\partial D}{\partial K} > 0.
\]

Increased turnover costs diminishes a worker’s willingness to engage in search.\textsuperscript{12} As search becomes more distant, low wage workers who are accumulating assets in order to eventually seek high wage employment become less willing to sacrifice today for more remote rewards. Low wage workers stay longer and consume more in low wage jobs. When they do switch to job search, they arrive prepared to search longer to offset the possibility of future turnover costs. They do so by arriving with higher assets and by consuming less given asset levels.

### Wages

The individual’s response to altering income are less transparent, with results available only for initial and terminal consumption. It is straightforward to establish that a pay rise in bad jobs increases consumption during low wage employment. Likewise, consumption at the end of high wage job search rises with low wages:

\[
\frac{\partial c_0}{\partial w_L} = \frac{\partial c_T}{\partial w_L} > 0;
\]

\[
\frac{\partial c_T}{\partial w_L} > 0
\]

(See the Appendix for details.)

\textsuperscript{11}Consumption as a function of the length or duration of search is in general ambiguous: $\frac{\partial c_t}{\partial K} \geq 0$. For low levels of $t$ this derivative is clearly positive as $\frac{\partial c_t=0}{\partial K} > 0$ but depending on risk aversion, the decline in consumption may be more rapid under higher turnover costs so that this derivative becomes negative.

\textsuperscript{12}If turnover costs are zero, workers would work at bad jobs and then search for infeasibly short periods. For $K = 0$, a “chattering” solution between employment and search results.
As low wage pay \( w_L \) does not affect the phase diagram, the new consumption path is a higher trajectory accompanied by a higher initial value. Consumption given assets rises \( \partial c(A_t) / \partial w_L > 0 \); however, given that \( c_0 \) has risen it is not possible to graphically determine whether assets at \( t = 0 \) are higher. Given a small rise in initial consumption on the new path, the duration of search will fall. For a sufficiently large rise we get the opposite effect. Given this ambiguity, it is not possible to tell (or so it appears) the effects on employment or search duration as well as asset accumulation.\(^{13}\) Although increased consumption reflects higher income from bad jobs, it is unclear how individuals alter the allocation of time between work and search.

When considering the individual’s response to changes in high wages, the analysis becomes less revealing. As pay in good jobs improves, initial as well as terminal consumption both decrease:

\[
\partial c_0 / \partial w_H = \partial c_T / \partial w_H < 0;
\]

With more attractive good jobs, low wage workers save at a higher rate in order to facilitate search. Now, however, the phase diagram shifts with \( w_H \) changes. For a given the terminal condition, there is higher consumption at each asset level - the consumption paths rotate upward. As such, little can be inferred regarding consumption while searching. Workers consume less toward the end of search activity (when assets are low) reflecting a the greater return to search. Consumption at the outset of search is also lower although the way in which consumption given relatively high assets responds is undetermined. Likewise, the steeper path is balanced by a fall in initial and terminal consumption so working out the duration of search and the initial asset level can not be done diagrammatically.

### Limited and Unlimited Duration UI Payments

Turning to direct policy instruments\(^ {14}\), the Appendix demonstrates that saving rates fall as UI payments rise:

\[
\partial c_0 / \partial b = \partial c_T / \partial b < 0
\]

\(^{13}\)Analytically, these effects depend on the solution of differential equation solution for \( c_t \). Given the structure of this differential equation, the outcome is likely to depend on third derivatives for \( u(c_t) \).

\(^{14}\)An alternative policy is to relax the credit market imperfection. As capital becomes more available, \( F \) increases and decision makers borrow until the new constraint is reached (For large increases in \( F \) or no capital constraint, borrowers will ultimately search only once and then take on low wage jobs permanently.) Like improved UI payment schemes, the worker benefits from this policy. Note, however, that when the new credit constraint binds, the worker is in a worse position and hence worse off. The worker responds with higher savings rates \((\partial c_0 / \partial F = \partial c_T / \partial F < 0)\) but as usual the remaining effects are elusive.
As one might expect by now, further effects are again limited. More importantly, such effects have limited appeal. Most unemployment insurance programs provide payments for only a limited duration.

Fortunately, in this model, incorporating UI payments for a fixed period is relatively uncomplicated. In particular, suppose UI payments $b$ last for a period of length $H$. After this period, payments stop until the worker quits job search and re-qualifies by going through a period of (low wage) employment. This policy has three possible outcome types.

- **Case 1** $T < H$. Here, the individual stops high wage job search before the termination of benefits. The budget constraint and hence the analysis are unaffected from before.

- **Case 2** $T > H$. In the second possible outcome, the individual continues to search after exhausting benefits. In this case, assets must be positive when the UI payments terminate in order to fund search beyond $H$. For this individual, the budget constraint (at the outset of high wage job search and for $w_F = 0$) is given by

$$A_D + \int_0^H (b - c_t)e^{-rt}dt - \int_H^T c_t e^{-rt}dt - e^{-rT} A_T \geq 0$$

where $A_D$ is found as before.

- **Case 3** $T = H$. If search ends when benefits terminate, the individual may choose either (i) to dovetail assets so that they equal zero after exactly $H$ periods or (ii) to exhaust assets before UI payments cease. (In (ii), the individual can at most consume $b$ in each period of search, an outcome preferred to low wage employment.) For this individual, the termination of search is in effect exogenous while the budget constraint is now

$$A_D + \int_0^H (b - c_t)e^{-rt}dt - e^{-rH} A_H \geq 0$$

Two policy changes are considered. The first reform involves a lengthening of the benefit duration $H$ while keeping the per period payment $b$ constant. The alternative reform also extends the payment period $H$ but simultaneously lowers the UI payment $b$ in such a way that the present value of benefits is unchanged.
If search ends when UI benefits run out (Case 3), increasing the duration of UI payments increases search one for one, regardless of any alteration in UI payments. Due to the liquidity constraint, UI benefits extend search by more than an equivalent lump sum payment. The timing of payments matters for these job seekers. This adjustment in the duration of search is fundamentally different from the response from both the Case 2 individual whose search outlasts the duration of benefits and the Case 1 job seeker who does not search until benefits run out.

For these unconstrained individuals \((T \neq H)\), both reforms involve (if any response) only a pure income effect. In particular, an increase in \(H\) alone has no effect in Case 1. It is trivial to see that extending unexhausted benefits does not alter behavior. On the other hand, in response a balanced budget policy change, this individual will, however, act as if there were a rise in \(b\). A contrasting picture emerges in Case 2. A policy that extends but does not change the benefit level generates more resources to allocate - search is more attractive. When the budget is balanced, there are no additional resource and hence no response.

When a Case 1 or Case 2 job seeker experiences more (real) UI resources, it seems likely that the duration of search will rise while the duration of low wage employment will decline. However, without complete comparative statics for \(T\), it is not possible fully characterize these results. At this point, it may be that when \((T \neq H)\) individuals extend search more than the increase in the duration of benefits. Although apparently unlikely, these job seekers may be more responsive than Case 3 individuals to the improved search conditions.

**Summary**

This paper examines job search and consumption behavior in an economy with high and low wage jobs. Workers, of course, prefer high wage employment but to secure one of these good jobs they must first engage in uncertain search. To finance consumption during search, job seekers eat into assets - debt financing is not available. Worker without assets can overcome this credit constraint (to some extent) by accepting low paying jobs. Such employment is readily available but hinders the ability to search for high pay work.

This paper demonstrates that individuals may work in “bad” jobs to accumulate assets which subsequently fund search for good jobs. If the ensuing search is unsuccessful, workers repeat their asset accumulation while in low wage employment. An s-S inventory rule emerges but in this case the stock of
assets is not run down by constant consumption. Instead, consumption falls as search continues until either a good job is found or assets are depleted.

As in a Harris-Todaro (1970) economy, workers trade off the benefits of immediately available low wage work against the those of unemployment while looking for good jobs. Here, however, there are explicit flows between sectors as workers move in and out of low wage employment. The size of these flows is determined by wages and the costs of moving across sectors. Low turnover costs generate rapid movements between high wage job search and low wage employment.

Since a necessary condition for job turnover is that the no-debt constraint must bind, such constraints have implications on the distribution of income. In the literature, it has been shown that borrowing constraints can affect the distribution of income by restricting human capital investment choices. In this analysis, capital market imperfections have further implications for the distribution of income. Without constraints workers are of course better off although ex post some will be unlikely and be resigned to low wage employment with debts. The observed income distribution changes.

In summary, this paper establishes that

- Voluntary planned quits occur in a cyclical pattern. This cycle provides an explanation for a series of short job durations (a low wages) followed by a long spell of employment at high wages.

- Consumption while searching falls over time until either a good job is found or assets run out and the worker accepts a low wage job. In contrast, consumption during employment in low wages is less than earnings and equal to consumption at the beginning of job search: $w_L > \xi = c_0 > c_T > 0$.

- The durations of job search and employment as well as the pattern of consumption and asset accumulation are related to the transaction costs $K$ and wages. Turnover costs critically affect the flows between sectors and the consumption of both employed and unemployed workers. Note as well that the effects from a rise in high wages qualitatively differs from the effects generated by a fall in low wage jobs. It is not just relative wages that matter.

- The employment response to unemployment insurance benefits depends critically on individual assets.
REFERENCES

References


APPENDIX

Claim 1 Proof : For any fixed $\bar{D}$ where $0 \less \bar{D} < \infty$, define

$$V_D(A_0; T) = \max_{\zeta_t, c_t} \int_0^{\bar{D}} u(\zeta_t)e^{-rt}dt + e^{-rD} \left\{ \int_0^T u(c_t)e^{-(r+\alpha)t}dt ight\}$$

$$+ \alpha \int_0^T u(rA_s + w_H)e^{-(r+\alpha)s}ds + e^{-(r+\alpha)T}u(rA_0 + w_L)/r \right\}$$

subject to

$$A_0 + \int_0^{\bar{D}} (w_L - \zeta_t)e^{-rt}dt - e^{-rD}\int_0^T c_t e^{-rt}dt - e^{-r(\bar{D}+T)}A_0 \geq 0$$

and

$$A_0 + \int_0^{\bar{D}} (w_L - \zeta_t)e^{-rt}dt - e^{-rD}\int_0^T c_t e^{-rt}dt - e^{-r(\bar{D}+T)}A_0 \geq 0$$
for $K = 0$. This is the worker’s basic decision problem but with exogenous $D,T$ and $A_T = A_0$. As such the same first order conditions (8) and (9) continue to apply along with the budget constraint.

To establish that it is optimal to search when $K$ is small, it is sufficient to establish that

$$\Delta = V_D(A_0; T) - u(rA_0 + w_L)/r > 0$$

for some $T > 0$. Evaluated at $T = 0$, $V_D(A_0; T)_{|T=0} = u(rA_0 + w_L)/r$ and hence $\Delta = 0$. Moreover, differentiation of $V_D(A_0; T)$ and the budget constraint evaluated at the optimal choices for $\zeta_t, c_t$ gives

$$d\Delta = e^{-rD-(r+\alpha)T} \left[ -u'(c_T)(c_T - rA_0) + u(c_T) + \frac{\alpha}{r}u(rA_0 + w_H) \\
- \frac{r + \alpha}{r}u(rA_0 + w_L) \right] dt$$

As $T \to 0$, $c_T \to \bar{c} = rA_0 + w_L$. Therefore

$$\lim_{T \to 0} \frac{d\Delta}{dT} = e^{-rD} \left[ \frac{\alpha}{r} (u(rA_0 + w_H) - u(rA_0 + w_L)) - u'(rA_0 + w_L)w_L \right] > 0$$

Therefore, $\Delta > 0$ for some $T > 0$. QED

Claim 2 Proof: Since $c_t > rA_t$, $\lim_{t \to \infty} A_T = 0$. Given $A = 0$, consumption equals zero during search: $c_t = 0$. For search not to be too attractive, it is sufficient to establish that at some point the worker must strictly prefer low wage employment over search with zero consumption. If the worker chooses to never move into low wage employment so that $T \to \infty$, the expected lifetime utility of having zero assets is given by

$$\frac{\alpha}{r} \int_0^T u(w_H)e^{-(r+\alpha)t}dt = \frac{\alpha}{r(r + \alpha)}u(w_H)$$

A worker with zero assets can always work indefinitely at a low wage job with payoff: $u(w_L)/r$. Given these options, the result follows. QED

Claim 3 Proof: To prove this claim, we first establish four intermediate results. Recall that assumptions A1 and A2 ensure that $0 < T < \infty$ and $D < \infty$. 

(ii) $e^{-r(D+T)}A_T \geq 0$
Lemma 4 \( V'(A_0) = u'(c_0) \)

Proof: This is a familiar envelope result. Total differentiation of \( V(A_0) \) gives

\[
V'(A_0)dA_0 = \frac{1 - e^{-rD}}{r} \left[ u'(\zeta) - \alpha \int_0^T u'(rA_s + w_H) e^{-\alpha s} ds \right] d\xi
\]

\[
+ e^{-rD} \int_0^T \left\{ u'(c_t)e^{-\alpha t} - \alpha \int_0^T u'(rA_s + w_H) e^{-\alpha s} ds \right\} dc_t e^{-rt} dt
\]

\[
+ e^{-rD} \left\{ u(\zeta) - r \int_0^T u(c_t)e^{-(r+\alpha)t} dt - \alpha \int_0^T u(rA_s + w_H) e^{-(r+\alpha)s} ds \right\} dD
\]

\[
- re^{(r+\alpha)T} V(A_T) + \alpha e^{rD} \left[ rA_0 + w_L - \zeta \right] \int_0^T u'(rA_s + w_H) e^{-\alpha s} ds \right\} dD
\]

\[
+ e^{-rD-(r+\alpha)T} \left[ u(c_T) + \frac{\alpha}{r} u(rA_T + w_H) - (r+\alpha)V(A_T) \right] dT
\]

\[
+ e^{-rD-(r+\alpha)T} V'(A_T)dA_T + \alpha \int_0^T u'(rA_s + w_H) e^{-\alpha s} ds dA_0
\]

Plugging in the first order conditions gives

\[
V'(A_0)dA_0 = \mu_1 \left\{ \frac{1 - e^{-rD}}{r} d\xi + e^{-rD} \int_0^T dc_t e^{-rt} dt + e^{-r(D+T)} [c_T - A_T] dT
\]

\[
- e^{-rD} \left[ w_L - \zeta + r \int_0^T c_t e^{-rt} dt + re^{rT} A_T + rK \right] dD + e^{-r(D+T)} dA_T
\]

\[
+ \mu_2 e^{-r(D+T)} dA_T + \alpha \int_0^T u'(rA_s + w_H) e^{-\alpha s} ds dA_0
\]
Likewise, differentiation of the budget constraint gives

$$dA_0 = \frac{1 - e^{-rD}}{r} d\xi + e^{-rD} \int_0^T dc_t e^{-rt} dt + e^{-r(D+T)} [c_T - A_T] dT$$

$$-e^{-rD} \left[ w_L - \bar{\xi} + r \int_0^T c_t e^{-rt} dt + re^{-rT} A_T + r K \right] dD + e^{-r(D+T)} dA_T$$

Combining gives

$$V'(A_0) dA_0 = \left\{ \mu_1 + \alpha \int_0^T u'(rA_s + w_H)e^{-\alpha s} ds \right\} dA_0 + \mu_2 e^{-r(D+T)} dA_T + e^{-rD-(r+\alpha)T} dV(A_T)$$

From the Kuhn Tucker condition in (7), it is straightforward to demonstrate that $\mu_2 e^{-r(D+T)} dA_T = 0$ and hence

$$V''(A_0) = \mu_1 + \alpha \int_0^T u'(rA_s + w_H)e^{-\alpha s} ds = u'(c_0)$$

from equation (8). QED

**Lemma 5** If $A_T > 0$, then $V''(A_T) < 0$

**Proof:** $A_T > 0$ implies that $\mu_2 = 0$ and therefore from the first order conditions

$$e^{-\alpha T} V'(A_T) = \mu_1 = e^{-\alpha T} u'(c_T).$$

Differentiation gives

$$V''(A_T) = u''(c_T) \frac{dc_T}{dA_T}.$$ 

Likewise differentiation of the first order condition

$$(r + \alpha) V(A_T) = u(c_T) + \frac{\alpha}{r} u(rA_T + w_H) + u'(c_T)(rA_T - c_T)$$

gives

$$(r + \alpha) V'(A_T) = \alpha u'(rA_T + w_H) + ru'(c_T) + u''(c_T)(rA_T - c_T) \frac{dc_T}{dA_T}.$$
Combining yields the desired result:

\[ V''(A_T) = \frac{\alpha [u'(c_T) - u'(rA_T + w_H)]}{rA_T - c_T} < 0. \]

QED

**Lemma 6** If \( D > 0 \), then \( V''(A_0) = 0 \)

**Proof**: If \( D > 0 \), then \( \bar{\xi} \) is well defined and satisfies

\[ rV(A_0) = u(\bar{\xi}) + u'(\bar{\xi})(rA_0 + w_L - \bar{\xi}). \]

Therefore,

\[ rV'(A_0) = ru'(\bar{\xi}) + u''(\bar{\xi})(rA_0 + w_L - \bar{\xi}) \frac{d\bar{\xi}}{dA_0} \]

From Lemma 4 and the result that \( \bar{\xi} = c_0 \), it follows that \( d\bar{\xi}/dA_0 = 0 \). Further, Lemma 4 yields

\[ V''(A_0) = u''(\bar{\xi}) \frac{d\bar{\xi}}{dA_0} = 0 \]

QED

**Lemma 7** Given \( A_0 > 0, A_T < A_0 \).

**Proof**: 

**Case 1**: \( D = 0 \). As \( 0 < T < \infty \) and \( c_t > rA_t \), \( A_T < A_0 \) follows immediately.

**Case 2**: \( D > 0 \). Suppose \( A_T \geq A_0 \). Define assets at the end of low wage employment by

\[ A_D = e^{rD}A_0 + \frac{e^{rD} - 1}{r}(w_L - \bar{\xi}). \]

Notice that \( A_D > A_T \) by the logic used in Case 1. Individuals with assets \( A \in [A_0, A_D) \) choose positive low wage employment, \( D > 0 \), and hence from Lemma 6, \( V''(A) = 0 \). As a result

\[ V'(A) = V'(\tilde{A}) \quad \forall \ A, \tilde{A} \in [A_0, A_D). \]

As \( 0 < T < \infty \), \( c_0 > c_t \). Moreover, \( A_T > 0 \) implies \( \mu_2 = 0 \) so that \( V'(A_T) = u'(c_T) \). From Lemma 1, we therefore get the contradiction that

\[ V'(A_T) = u'(c_T) > u'(c_0) = V'(A_0). \]

QED

These lemmas are now used to establish the claim. Suppose \( A_T > 0 \). By Lemma 5, \( V''(A_T) < 0 \). By Lemma 7, \( A_T < A_0 \). From Lemma 6 and Claim 1, \( V''(A_T) = 0 \), a contradiction. QED
Comparative Static Calculations

For the analysis here let $A_0 = A_T = -F = 0$. These asset levels although optimal can be treated as exogenous in which case equations (14), and (16) become obsolete. At this point it is useful to define $V$, the value of starting (and completing) a cycle of low wage work followed by search for high wage employment where initial (and terminal) assets equal zero, $A_0 = 0$ (and $A_T = 0$):

$$V = 1 - \frac{e^{-rD}}{r}u(\bar{\zeta}) + e^{-rD} \left\{ \int_0^T u(c_t)e^{-(r+\alpha)t}dt \right\}$$

$$+ \frac{\alpha}{r} \int_0^T u(rA_s + w_H + w_F)e^{-(r+\alpha)s}ds + e^{-(r+\alpha)T}V$$

(17)

where the choice variables, $\bar{\zeta}, c_t, T$ and $D$ equal their optimal values as defined by the first order conditions (8), (9), (12), (13) and (15).

Rearranging the equations produces separate equations for $\bar{\zeta}$ and $c_T$ as functions of $V$ and exogenous parameters

$$rV - u(\bar{\zeta}) - u'(\bar{\zeta})[w_F + w_L - \bar{\zeta}] = 0$$

(18)

$$u(c_T) + \frac{\alpha}{r}u(w_H + w_F) - (r + \alpha)V + u'(c_T)(w_F + b - c_T) = 0$$

(19)

As shown in Lemma 4 (as part of the proof to Claim 3),

$$\mu_1 + \alpha \int_0^T u'(rA_s + w_H + w_F)e^{-\alpha s}ds = u'(\bar{\zeta})$$

To ease notation assume that $b = w_F = 0$. Differentiation reveals that after some manipulation:

$$[1 - e^{-rD-(r+\alpha)T}]dV = -e^{-rD}u'(\bar{\zeta}) \cdot dK + \frac{1 - e^{-rD}}{r}u'(\bar{\zeta}) \cdot dw_L$$

$$+ e^{-rD} \frac{\alpha}{r} \int_0^T u'(rA_s + w_H)e^{-(r+\alpha)s}ds \cdot dw_H$$

From (18),

$$rdV - u''(\bar{\zeta})[w_L - \bar{\zeta}] \cdot d\bar{\zeta} - u'(\bar{\zeta}) \cdot dw_L = 0$$

24
Plugging in for \(rdV\) gives

\[
d\xi = \frac{1}{1 - e^{-rD-(r+\alpha)T}} u''(\xi) \left[ w_L - \xi \right] \left\{ -re^{-rD} u'(\xi) \cdot dK \\
- e^{-rD} (1 - e^{-(r+\alpha)T}) u'(\xi) \cdot dw_L + e^{-rD} \alpha \int_0^T u'(rA_s + w_H) e^{-(r+\alpha)s} ds \cdot dw_H \right\}
\]

Likewise, from (19),

\[
\frac{\alpha}{r} u'(w_H) \cdot dw_H - (r + \alpha) \cdot dV - u''(c_T)c_T \cdot dc_T = 0.
\]

Again plugging in for \(dV\) gives

\[
dc_T = \frac{-(r + \alpha)}{r [1 - e^{-rD-(r+\alpha)T}] u''(c_T)c_T} \left\{ re^{-rD} u'(\xi) \cdot dK \\
- (1 - e^{-rD}) u'(\xi) \cdot dw_L + \alpha \frac{1 - e^{-rD}}{r + \alpha} u'(w_H) + \\
e^{-rD} \int_0^T \left[ u'(w_H) - u'(rA_s + w_H) \right] e^{-(r+\alpha)s} ds \cdot dw_H \right\}
\]